EE 434 Electromagnetic Waves, Spring 2009
Exam #2, 2009/3/20 Solutions

(1) The reflection coefficient is

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}$$

and

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

Since $\mu = \mu_0$, $\epsilon_1 = \epsilon_0$, and $\epsilon_2 = 3\epsilon_0$, we get

$$\eta_1 = \sqrt{\frac{4\pi \times 10^{-7}}{8.854 \times 10^{-12}}} = 377\,\Omega$$

$$\eta_2 = \sqrt{\frac{4\pi \times 10^{-7}}{3 \times 8.854 \times 10^{-12}}} = 218\,\Omega$$

The reflection coefficient is thus

$$\Gamma = \frac{218 - 377}{377 + 218} = -0.267$$

The amount of power reflected is

$$P_r = P\Gamma^2 = 100 \times 0.267^2 = 7.14\,\frac{\mathrm{W}}{\mathrm{m}^2}$$

(2) We use Snell's law,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$n_1 = 1$, and $n_2 = \sqrt{\epsilon_{2r}\mu_{2r}} = \sqrt{\epsilon_{2r}}$ since the material is non-magnetic. Also, $\eta_2 = \sqrt{\mu_0/\epsilon_0\epsilon_{2r}}$, so

$$\epsilon_{2r} = \frac{\mu_0}{\epsilon_0 \eta_2^2} = \frac{4\pi \times 10^{-7}}{8.854 \times 10^{-12} \times 200^2} = 3.548$$

and thus

$$n_2 = \sqrt{\epsilon_{2r}} = \sqrt{3.548} = 1.884$$

Now from Snell's law we get

$$\theta_t = sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1}{1.884}\sin 45^\circ\right) = 22.04^\circ$$

(3) The reflection coefficient for parallel light is

$$\Gamma_{\parallel} = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_t + \eta_2 \cos\theta_i}$$

and $\eta_1 = 377\,\Omega$, and $\eta_2 = 200\,\Omega$, so we get

$$\Gamma_{\parallel} = \frac{377\cos 45^\circ - 200\cos 22.04^\circ}{377\cos 22.04^\circ + 200\cos 45^\circ} = 0.1654$$

so 16.54% of the electric field is reflected.

(4) We know that $Z_3 = \eta_3$, so $z_3 = 1$. Next, $Z_2(0) = Z_3(0)$, so

$$z_2(0) = \frac{\eta_3}{\eta_2} z_3 = \frac{150}{50} = 3$$

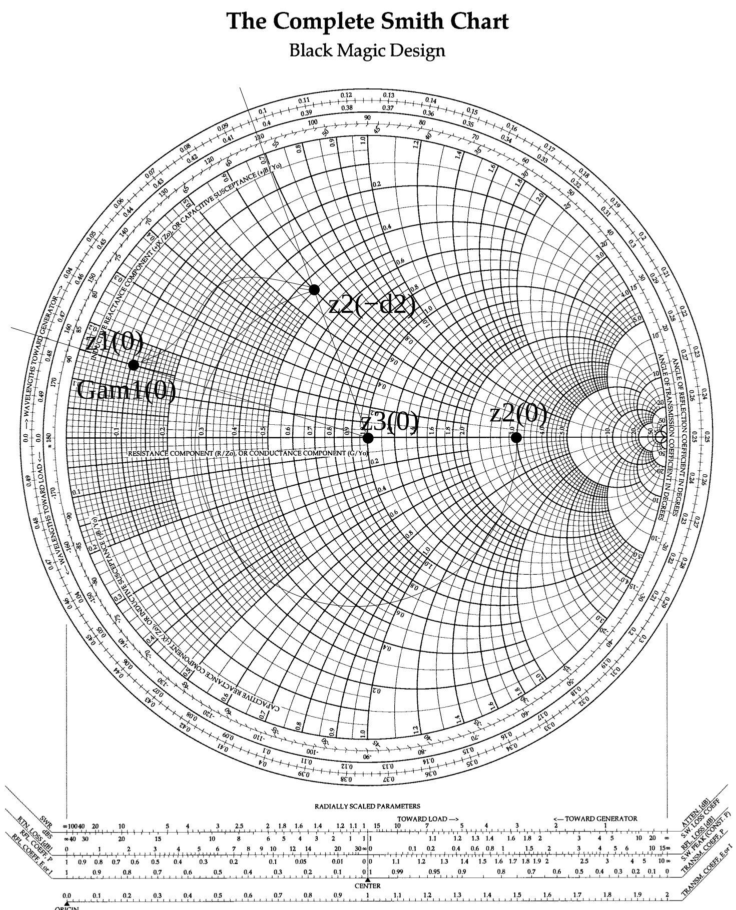
Next we rotate and read $z_2(-d_2) = 0.45 + 0.60j$. Then,

$$z_1(0) = \frac{\eta_2}{\eta_1} Z_2(-d_2) = \frac{50}{200} z_2(-d_2) = 0.105 + 0.15j$$

That point is also the reflection coefficient in medium 1 at the 1-2 interface. Reading it off it looks roughly like

$$\Gamma_1(0) = 0.8e^{j163^\circ}$$

The Smith chart is included below



(5) We know that for non-conducting, non-magnetic materials, $n = \sqrt{\epsilon_r}$, so $\epsilon_r = n^2$. The Brewster angle is given by

$$\tan\theta_B = \sqrt{\frac{\epsilon_{2r}}{\epsilon_{1r}}} = \sqrt{\frac{n_2^2}{n_1^2}}$$

so

$$\theta_B = \tan^{-1}\sqrt{\frac{n_2^2}{n_1^2}} = \tan^{-1}\sqrt{\frac{1.8^2}{1.2^2}} = 56.31°$$

The critical angle for total internal reflection is the incidence angle which gives a transmission angle of 90°, so if we assume $\theta_1 = 90°$, we write

$$n_2 \sin\theta_c = n_1 \sin 90°$$

$$\theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right) = \sin^{-1}\left(\frac{1.2}{1.8}\right) = 41.81°$$