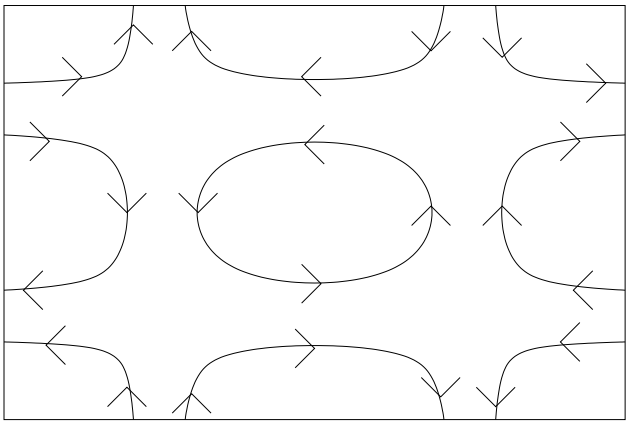
EE 434 Electromagnetic Waves, Spring 2009
Exam #4, 2009/5/8
Solution

Questions

(1) Sketch the electric field of the TE_{22} mode in a rectangular waveguide.



(2) Starting with the definition of the index of refraction, $n = c/v$, derive the expression for n in terms of the relative permittivity and relative permeability. If $\mu_1 = \mu_2$ and $\epsilon_{1r} = 1$ and $\epsilon_{2r} = 3$, what is the value of the critical angle for total internal reflection?

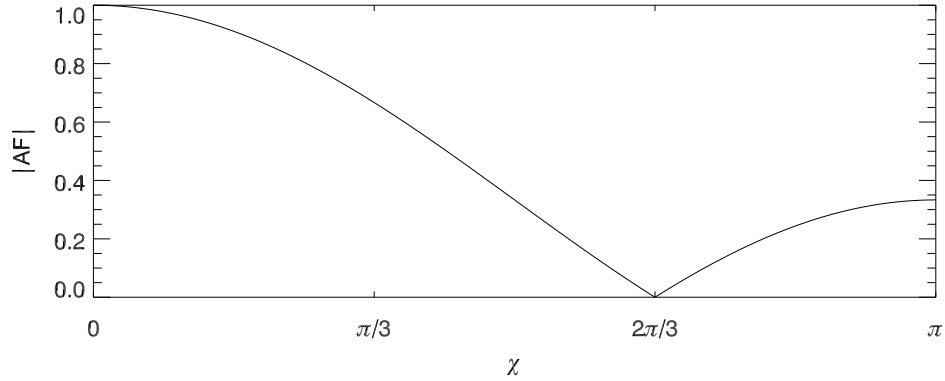
$$n = \frac{c}{v} = \frac{\sqrt{\epsilon\mu}}{\sqrt{\epsilon_0\mu_0}} = \sqrt{\epsilon_r\mu_r}$$

The critical angle for total internal reflection is the angle in medium 2 which causes a refraction angle of $\theta_1 = 90^\circ$ in medium 1.

$$n_1 \sin\, 90^\circ = n_2 \sin\theta_2$$

$$\theta_2 = \sin^{-1}\frac{n_1}{n_2} = \sin^{-1}\sqrt{\frac{\epsilon_{1r}}{\epsilon_{2r}}} = \sin^{-1}\frac{1}{\sqrt{3}} = 35.3^\circ$$

(3) The Array function, $AF(\chi)$ (and $\chi = \beta d\cos\phi - \psi$) looks like this for $N = 3$:



Now, if $d = \lambda/2$, what value of ψ will cause a maximum in the array factor in the forward $\phi = 0$ direction? And what is then the value of the array factor in

the reverse $\phi = \pi$ direction? Sketch the array factor in polar coordinates (the usual plot).
The array factor is maximum at $\chi = 0$, so we need

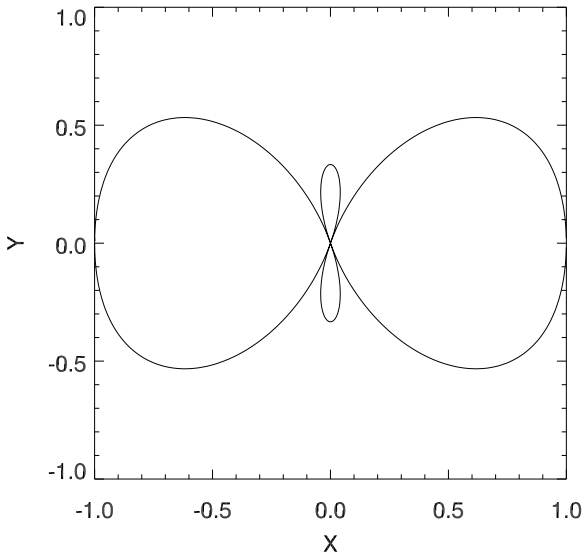
$$\beta d \cos\phi - \psi = 0$$

for $\phi = 0$

$$\beta d - \psi = 0$$

$$\psi = \beta d = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi$$

For these values of d and ψ, we have for $\phi = \pi$ that $\chi = \pi\cos\pi - \pi = -2\pi = 0$. The value of the array factor is thus the same in the reverse direction as in the forward direction. That means that the array factor plotted in the figure must map in $\phi \in \left[0; \frac{\pi}{2}\right]$ range, and the array factor plotted in ploar coordinates looks like this:



(4) An empty rectangular waveguide with $a = 5\,\text{cm}$ and $b = 3\,\text{cm}$ is excited with a frequency $f = 5.1\,\text{GHz}$. What modes can propagate at this frequency?
The critical frequency for mode mn is

$$f_{c,mn} = \frac{1}{2\pi\sqrt{\mu_0\epsilon_0}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Now we calculate using $\mu_0 = 4 \times \pi \times 10^{-7}$, $\epsilon_0 = 8.854 \times 10^{-12}$, $a = 0.05$, and $b = 0.03$

f_c (GHz)	m		
n	0	1	2
0		3.00	6.00
1	5.00	5.83	7.81
2	9.99	10.43	11.65

Only 10 and 01 mode can propagate at that frequency. There are no TM modes, so we have only TE_{01} and TE_{10}.