

EE 434 Electricity and Magnetism, Spring 2009

Homework #8 Solution

6.19

Begin with Snell's law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Assume that $n_1 = 1$, and $n_2 = A + \frac{B}{\lambda^2}$, we get

$$\sin \theta_1 = \left(A + \frac{B}{\lambda^2} \right) \sin \theta_2$$

and

$$\theta_2 = \sin^{-1} \left[\frac{\sin \theta_1}{\left(A + \frac{B}{\lambda^2} \right)} \right]$$

Now using the given values we get

color	wavelength (nm)	refraction angle (degrees)
violet	400	19.058
blue	450	19.144
green	550	19.251
yellow	600	19.286
orange	650	19.313
red	700	19.334

6.20

(a) We use Snell's law at the 1-2 interface

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\theta_2 = \sin^{-1} \left(\frac{n_1}{n_2} \sin \theta_1 \right)$$

Next we use Snell's law at the 2-3 interface

$$n_2 \sin \theta_2 = n_3 \sin \theta_3$$

$$\begin{aligned} \theta_3 &= \sin^{-1} \left(\frac{n_2}{n_3} \sin \theta_2 \right) \\ &= \sin^{-1} \left(\frac{n_2 n_1}{n_3 n_2} \sin \theta_1 \right) \\ &= \sin^{-1} \left(\frac{n_1}{n_3} \sin \theta_1 \right) \end{aligned}$$

(b) We are looking for the value of θ_1 which will make $\theta_3 = 90^\circ$. That is

$$90^\circ = \sin^{-1} \left(\frac{n_1}{n_3} \sin \theta_1 \right)$$

$$\sin \theta_1 = \frac{n_3}{n_1}$$

$$\theta_1 = \sin^{-1} \left(\frac{n_3}{n_1} \right)$$

(c) It does not differ.

(d) To have total internal reflection at the first interface we require that $\theta_2 = 90^\circ$, which means

$$90^\circ = \sin^{-1} \left(\frac{n_2}{n_1} \sin \theta_1 \right)$$

$$\theta_1 = \sin^{-1} \left(\frac{n_2}{n_1} \right)$$