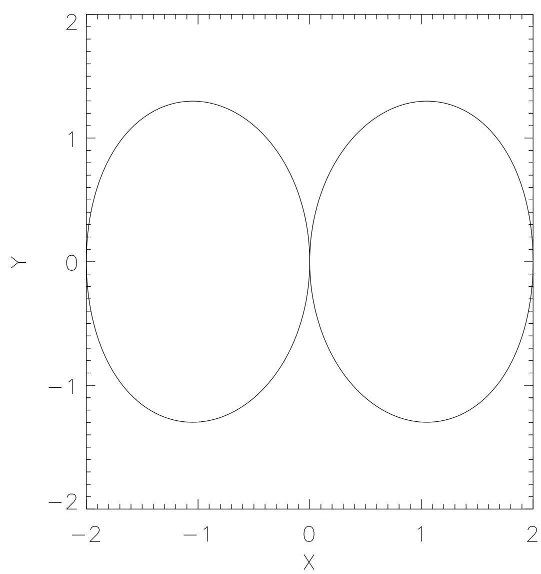
EE 434 Electricity and Magnetism, Spring 2009
Homework #12 Solution

9.7
The antenna factor is

$$\mathrm{AF} = \frac{\sin\left(\frac{N}{2}\left[\beta d\cos\phi - \psi\right]\right)}{\sin\left(\frac{1}{2}\left[\beta d\cos\phi - \psi\right]\right)}$$

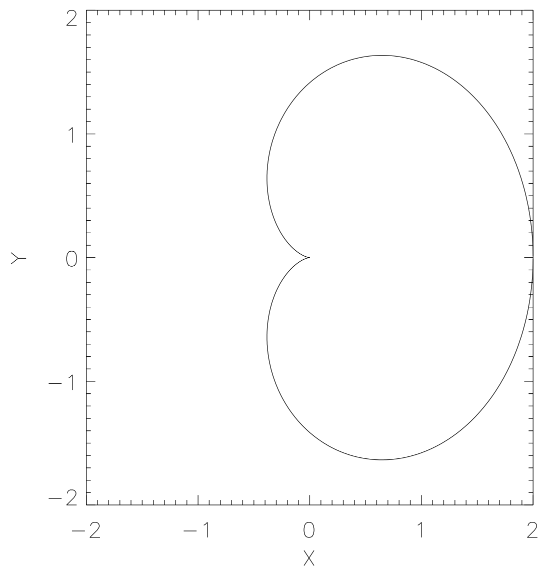
(a)

$$\begin{aligned}\mathrm{AF} =& \frac{\sin\left(\frac{2}{2}\left[\frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\phi - \pi\right]\right)}{\sin\left(\frac{1}{2}\left[\frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\phi - \pi\right]\right)} \\ =& \frac{\sin\left(\pi\cos\phi - \pi\right)}{\sin\left(\frac{1}{2}\left[\pi\cos\phi - \pi\right]\right)}\end{aligned}$$



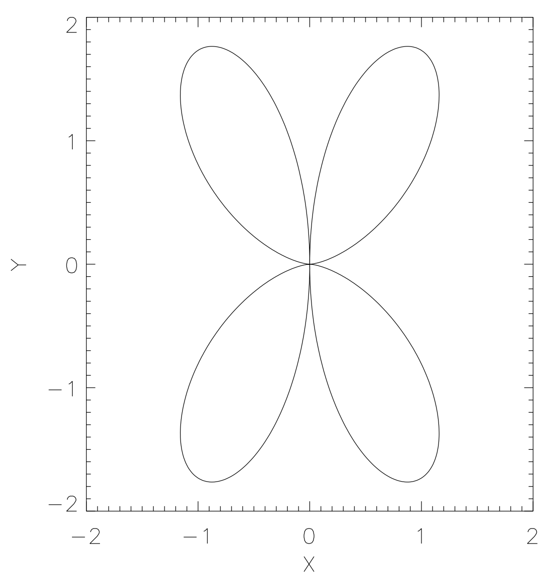
(b)

$$\begin{aligned}\mathrm{AF} =& \frac{\sin\left(\frac{2}{2}\left[\frac{2\pi}{\lambda}\frac{\lambda}{4}\cos\phi - \frac{\pi}{2}\right]\right)}{\sin\left(\frac{1}{2}\left[\frac{2\pi}{\lambda}\frac{\lambda}{4}\cos\phi - \frac{\pi}{2}\right]\right)} \\ =& \frac{\sin\left(\frac{\pi}{2}\cos\phi - \frac{\pi}{2}\right)}{\sin\left(\frac{1}{2}\left[\frac{\pi}{2}\cos\phi - \frac{\pi}{2}\right]\right)}\end{aligned}$$



(c)

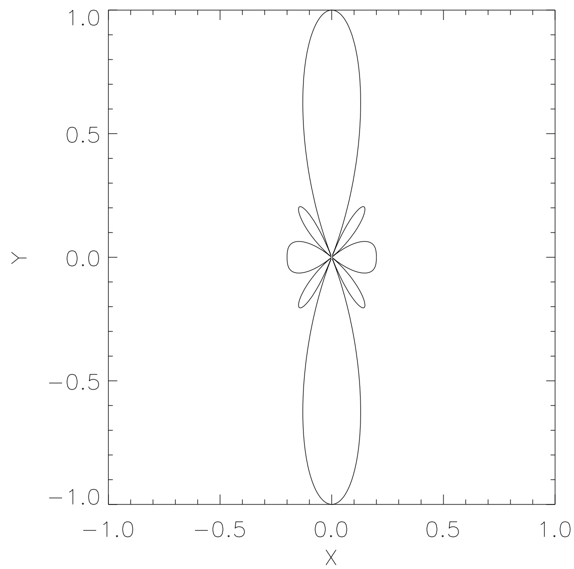
$$\mathrm{AF} = \frac{\sin\left(\frac{2}{2}\left[\frac{2\pi}{\lambda}\lambda\cos\phi - \pi\right]\right)}{\sin\left(\frac{1}{2}\left[\frac{2\pi}{\lambda}\lambda\cos\phi - \pi\right]\right)}$$
$$= \frac{\sin\left(2\pi\cos\phi - \pi\right)}{\sin\left(\frac{1}{2}\left[2\pi\cos\phi - \pi\right]\right)}$$



9.8

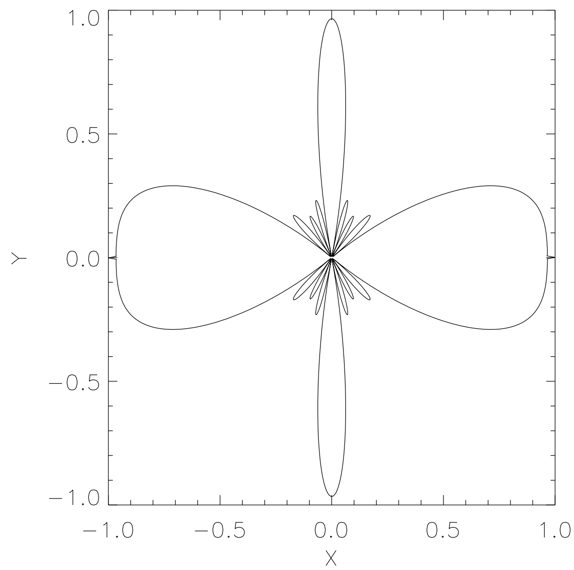
(a) Broad side array with $N = 5$, and $d = \lambda/2$. For a broad-side array, $\psi = 0$.

$$
\begin{aligned}
\mathrm{AF} =& \frac{\sin\left(\frac{5}{2}\frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\phi\right)}{\sin\left(\frac{1}{2}\frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\phi\right)} \\
=& \frac{\sin\left(\frac{5}{2}\pi\cos\phi\right)}{\sin\left(\frac{\pi}{2}\cos\phi\right)}
\end{aligned}
$$



(b) Broad side array with $N = 5$ and $d = \lambda$. For a board side array, $\psi = 0$.

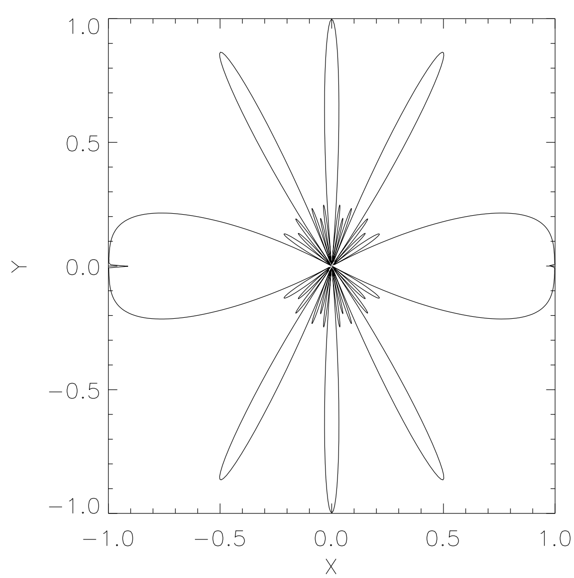
$$
\begin{aligned}
\mathrm{AF} =& \frac{\sin\left(\frac{5}{2}\frac{2\pi}{\lambda}\lambda\cos\phi\right)}{\sin\left(\frac{1}{2}\frac{2\pi}{\lambda}\lambda\cos\phi\right)} \\
=& \frac{\sin\left(5\pi\cos\phi\right)}{\sin\left(\pi\cos\phi\right)}
\end{aligned}
$$



(c) End-fire array with $N = 5$, and $d = 2\lambda$. I will use $\psi = \beta d$.

$$\beta d = \frac{2\pi}{\lambda} 2\lambda = 4\pi$$

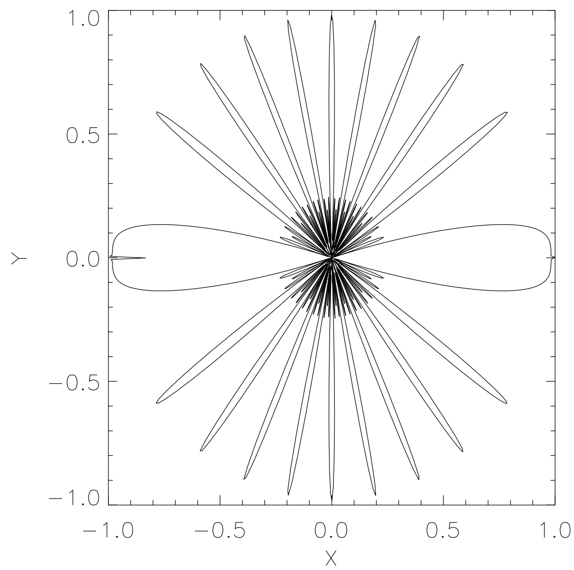
$$\text{AF} = \frac{\sin\left(10\pi\cos\phi - 10\pi\right)}{\sin\left(2\pi\cos\phi - 2\pi\right)}$$



End-fire array with $N = 5$ and $d = 5\lambda$. I will use $\psi = \beta d$.

$$\beta d = \frac{2\pi}{\lambda} 5\lambda = 10\pi$$

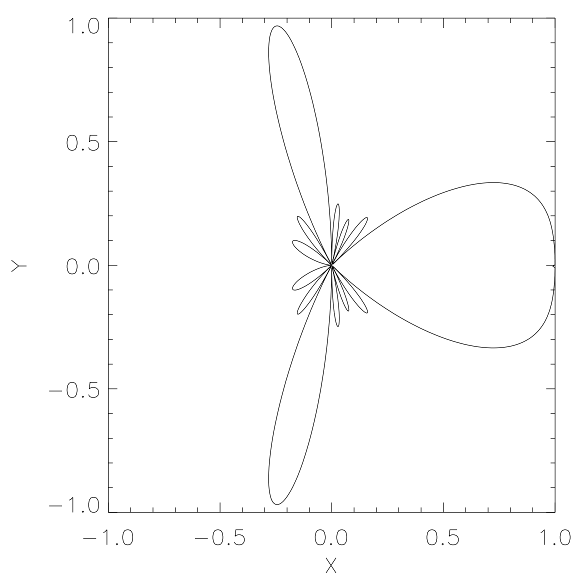
$$\mathrm{AF} = \frac{\sin\left(25\pi\cos\phi - 25\pi\right)}{\sin\left(5\pi\cos\phi - 5\pi\right)}$$



End-fire array with $d = 0.8\lambda$. I will use $\psi = \beta d$

$$\beta d = \frac{2\pi}{\lambda} 0.8\lambda = 1.6\pi$$

$$\mathrm{AF} = \frac{\sin\left(4\pi\cos\pi - 4\phi\right)}{1.6\pi\cos\phi - 1.6\pi}$$

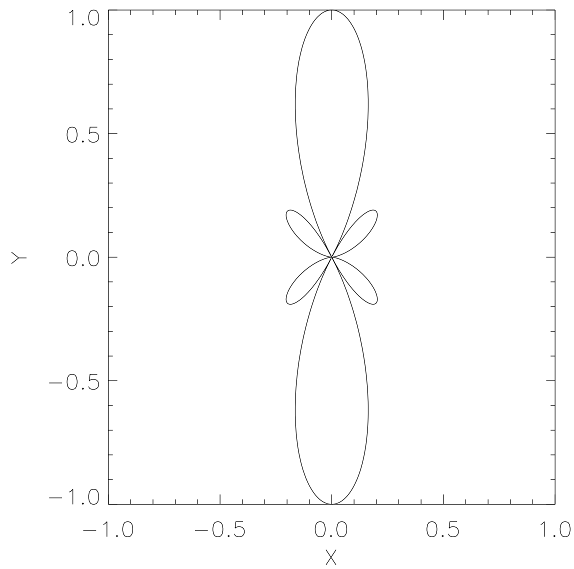


9.9

(a) Broadside means that $\psi = 0$. With $d = \lambda/2$ we have

$$\beta d = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi$$

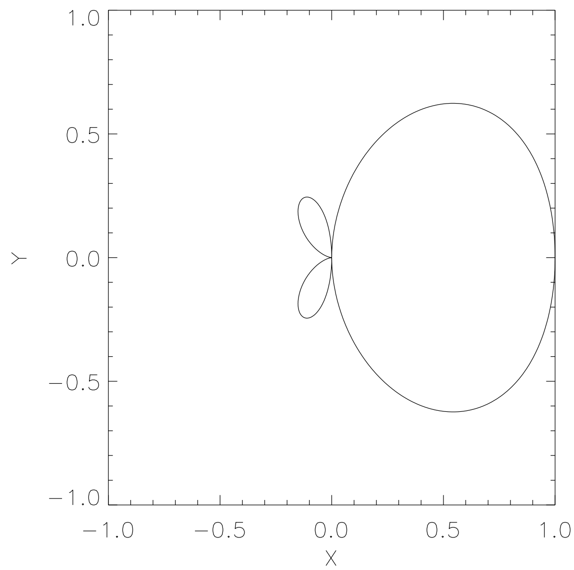
$$\text{AF} = \frac{\sin\left(2\pi\cos\phi\right)}{\sin\left(\frac{\pi}{2}\cos\phi\right)}$$



(b) $d = \lambda/4$, and $\psi = \frac{\pi}{2}$.

$$\beta d = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

$$\text{AF} = \frac{\sin\left(\pi\cos\phi - \pi\right)}{\sin\left(\frac{\pi}{4}\cos\phi - \frac{\pi}{4}\right)}$$



1.0
0.5
0.0
−0.5
−1.0
Y
−1.0
−0.5
0.0
0.5
1.0
X