EE 434 Electromagnetic Waves, Spring 2010
Exam 2 April 5, 2010
Equations

$$\epsilon_0 = 8.854 \times 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}} \qquad \mu = 4\pi \times 10^{-7}\,\frac{\mathrm{H}}{\mathrm{m}}$$

$$E_x = \frac{1}{\gamma^2 + \omega^2\mu\epsilon}\left(-j\omega\mu\frac{\partial H_z}{\partial y} - \gamma\frac{\partial E_z}{\partial x}\right) \qquad E_y = \frac{1}{\gamma^2 + \omega^2\mu\epsilon}\left(j\omega\mu\frac{\partial H_z}{\partial x} - \gamma\frac{\partial E_z}{\partial y}\right)$$

$$H_x = \frac{1}{\gamma^2 + \omega^2\mu\epsilon}\left(j\omega\epsilon\frac{\partial E_z}{\partial y} - \gamma\frac{\partial H_z}{\partial x}\right) \qquad H_y = \frac{1}{\gamma^2 + \omega^2\mu\epsilon}\left(-j\omega\epsilon\frac{\partial E_z}{\partial x} - \gamma\frac{\partial H_z}{\partial y}\right)$$

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \gamma^2 + \omega^2\mu\epsilon\right]_{H_z}^{E_z} = 0 \qquad \frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \gamma^2 + \omega^2\mu\epsilon = 0$$

$$E_z = E_0 \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{j\omega t - \gamma z} \qquad H_z = H_0 \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{j\omega t - \gamma z}$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon} \qquad \omega_{c,mn} = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\beta = \omega\sqrt{\mu\epsilon}\sqrt{1 - \left(\frac{\omega_{c,mn}}{\omega}\right)^2} \qquad \gamma^2 + \omega^2\mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\eta_{\mathrm{TM}_{mn}} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \sqrt{\frac{\mu}{\epsilon}}\sqrt{1 - \left(\frac{\omega_{c,mn}}{\omega}\right)^2} \qquad \eta_{\mathrm{TE}_{mn}} = \sqrt{\frac{\mu}{\epsilon}}\frac{1}{\sqrt{1 - \left(\frac{\omega_{c,mn}}{\omega}\right)^2}}$$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}}\frac{1}{\sqrt{1 - \left(\frac{\omega_{c,mn}}{\omega}\right)^2}} \qquad v_g = \frac{\partial\omega}{\partial\beta} = \left(\frac{\partial\beta}{\partial\omega}\right)^{-1} = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{1 - \left(\frac{\omega_{c,mn}}{\omega}\right)^2}$$

$$\vec{P} = \vec{E} \times \vec{H} \qquad E = \int_0^a \int_0^b P_z\,dy\,dx$$