EE 434 Electromagnetic Waves, Spring 2010
Exam 4 May 11, 2010
Equations

$$\epsilon_0 = 8.854 \times 10^{-12}\ \frac{\mathrm{F}}{\mathrm{m}} \qquad \mu_0 = 4\pi \times 10^{-7}\ \frac{\mathrm{H}}{\mathrm{m}}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{E_x^+}{H_y^+} = -\frac{E_x^-}{H_y^-} \qquad \Gamma = \frac{E_{x1}^-}{E_{x1}^+} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} \qquad \tau = \frac{E_{x2}^+}{E_{x1}^+} = \frac{2\eta_2}{\eta_1 + \eta_2}$$

$$Z = \frac{E^+ + E^-}{H^+ + H^-} = \eta\frac{1+\Gamma}{1-\Gamma} \qquad \Gamma = \frac{E^-}{E^+} = \frac{Z-\eta}{Z+\eta}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \sin\theta_c = \frac{n_2}{n_1} \qquad \tan^2\theta_B = \frac{\epsilon_2}{\epsilon_1}$$

$$\Gamma_\parallel = -\frac{E_\parallel^r}{E_\parallel^i} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \qquad \Gamma_\perp = \frac{E_\perp^r}{E_\perp^i} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$E_x = \frac{1}{\gamma^2 + \omega^2\mu\epsilon}\left(-j\omega\mu\frac{\partial H_z}{\partial y} - \gamma\frac{\partial E_z}{\partial x}\right) \qquad E_y = \frac{1}{\gamma^2 + \omega^2\mu\epsilon}\left(j\omega\mu\frac{\partial H_z}{\partial x} - \gamma\frac{\partial E_z}{\partial y}\right)$$

$$H_x = \frac{1}{\gamma^2 + \omega^2\mu\epsilon}\left(j\omega\epsilon\frac{\partial E_z}{\partial y} - \gamma\frac{\partial H_z}{\partial x}\right) \qquad H_y = \frac{1}{\gamma^2 + \omega^2\mu\epsilon}\left(-j\omega\epsilon\frac{\partial E_z}{\partial x} - \gamma\frac{\partial H_z}{\partial y}\right)$$

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \gamma^2 + \omega^2\mu\epsilon\right]_{H_z}^{E_z} = 0 \qquad \frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \gamma^2 + \omega^2\mu\epsilon = 0$$

$$E_z = E_0 \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{j\omega t - \gamma z} \qquad H_z = H_0 \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{j\omega t - \gamma z}$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon} \qquad \omega_{c,mn} = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\beta = \omega\sqrt{\mu\epsilon}\sqrt{1 - \left(\frac{\omega_{c,mn}}{\omega}\right)^2} \qquad \gamma^2 + \omega^2\mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\eta_{\mathrm{TM}_{mn}} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \sqrt{\frac{\mu}{\epsilon}}\sqrt{1 - \left(\frac{\omega_{c,mn}}{\omega}\right)^2} \qquad \eta_{\mathrm{TE}_{mn}} = \sqrt{\frac{\mu}{\epsilon}}\frac{1}{\sqrt{1 - \left(\frac{\omega_{c,mn}}{\omega}\right)^2}}$$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}}\frac{1}{\sqrt{1 - \left(\frac{\omega_{c,mn}}{\omega}\right)^2}} \qquad v_g = \frac{\partial\omega}{\partial\beta} = \left(\frac{\partial\beta}{\partial\omega}\right)^{-1} = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{1 - \left(\frac{\omega_{c,mn}}{\omega}\right)^2}$$

$$\vec{P} = \vec{E} \times \vec{H} \qquad E = \int_0^a \int_0^b P_z \, dy \, dx$$

$$\vec{H} = \frac{I\,dl}{4\pi} \sin\theta \left(\frac{j\beta}{r} + \frac{1}{r^2} \right) e^{-j\beta r} \hat{\phi}$$

$$\vec{E} = \frac{j\eta I dl}{2\pi\beta} \left[\cos\theta \left(\frac{j\beta}{r^2} + \frac{1}{r^3} \right) \hat{r} - \frac{1}{2} \sin\theta \left(-\frac{\beta^2}{r} + \frac{j\beta}{r^2} + \frac{1}{r^3} \right) \hat{\theta} \right] e^{-j\beta r}$$

$$\vec{P}_{\text{avg}} = \frac{1}{2} Re \left[\vec{E}_0 \times \vec{H}_0^* \right] \qquad \vec{P}_{\text{avg}} = \frac{\eta}{2} \left(\frac{\beta I \, dl \sin\theta}{4\pi r} \right)^2 \hat{r} \qquad P_{\text{tot}} = \frac{\eta \beta^2 I^2 \, dl^2}{12\pi}$$

$$\vec{H} = \frac{jI_0}{2\pi \sin\beta_0 l} \frac{e^{-j\beta_0 r}}{r} F(\theta) \, \hat{\phi} \qquad \vec{E} = \frac{j\eta_0 I_0}{2\pi \sin\beta_0 l} \frac{e^{-j\beta_0 r}}{r} F(\theta) \, \hat{\theta}$$

$$F(\theta) = \frac{\cos(\beta_0 l \cos\theta) - \cos(\beta_0 l)}{\sin\theta}$$

$$E = V_0 F(\theta, \phi) \sum_{i=0}^{N-1} \frac{e^{-j(\beta r_i + \psi_i)}}{r_i} \qquad AF = \frac{1}{N} \frac{\sin\left(N \frac{\chi}{2}\right)}{\sin\left(\frac{\chi}{2}\right)} \qquad \chi = \beta d \cos\phi - \psi$$

$$D = \frac{4\pi \, I}{P_{\text{tot}}} \qquad D_0 = \frac{4\pi \, I_{\max}}{P_{\text{tot}}}$$