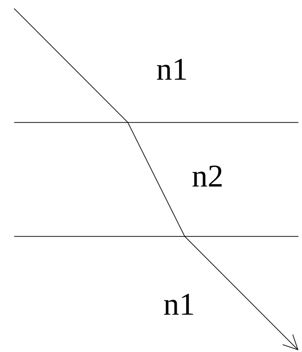
EE 434 Electromagnetic Waves, Spring 2010
Exam 1 March 1, 2010
solution

Rules: This is a open book test. You may use the textbook as well as your notes. The exam will last 50 minutes. Each problem counts equally toward your grade.

1. **A dielectric slab with parallel sides and index n_2 is embedded in a material of index n_1, as shown ($n_2 > n_1$). Show that a wave will exit the slab at the same angle as it enters it, as shown in the figure.**



At the first interface we have

$$n_1 \sin\theta_{i1} = n_2 \sin\theta_{t2}$$

and at the second interface we have

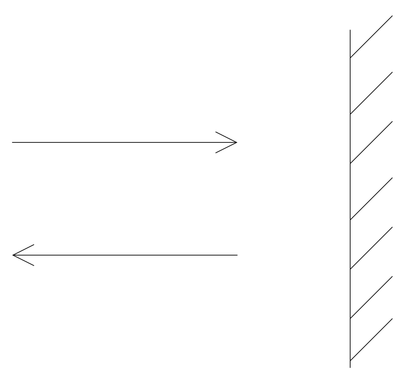
$$n_2 \sin\theta_{i2} = n_1 \sin\theta_{t3}$$

Since $\theta_{t2} = \theta_{i2}$, we combine the two equations to give us

$$n_1 \sin\theta_{i1} = n_1 \sin\theta_{t3}$$

$$\theta_{i1} = \theta_{t3}$$

2. **A plane wave in air at a frequency of 100 MHz is normally incident onto a perfectly conducting surface, reflecting and creating a standing wave. Not counting the null at the surface, give the distance from the surface to the second null in the electric field as well as in the magnetic field.**



First, compute the wavelength,

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{100 \times 10^6} = 3\,\mathrm{m}$$

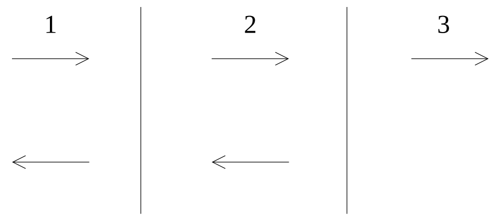
The standing wave electric field has nulls every half wavelength starting at the surface of the conductor. The second null, not counting the one at the surface is thus located at a distance

$$d_2 = 2\frac{\lambda}{2} = \lambda = 3\,\mathrm{m}$$

The standing wave magnetic field has nulls every half wavelength starting a quarter wavelength from the conductor. The second null is thus located at a distance

$$d_2 = \frac{\lambda}{4} + \lambda 2 = \frac{3}{4}\lambda = \frac{3}{4} \times 3 = 2.25\,\mathrm{m}$$

3. **Three dielectric regions bound each other. Their impedances are $\eta_1 = 100\,\Omega$, $\eta_2 = 150\,\Omega$, and $\eta_3 = 50\,\Omega$. Dielectrics 1 and 3 extend to infinity, and dielectric 2 has thickness $d_2 = \frac{\lambda_2}{8}$. Use a Smith chart to determine the reflection coefficient in medium 1 at the 1-2 interface, drawing your derivation on the Smith chart and labeling it as discussed in class.**

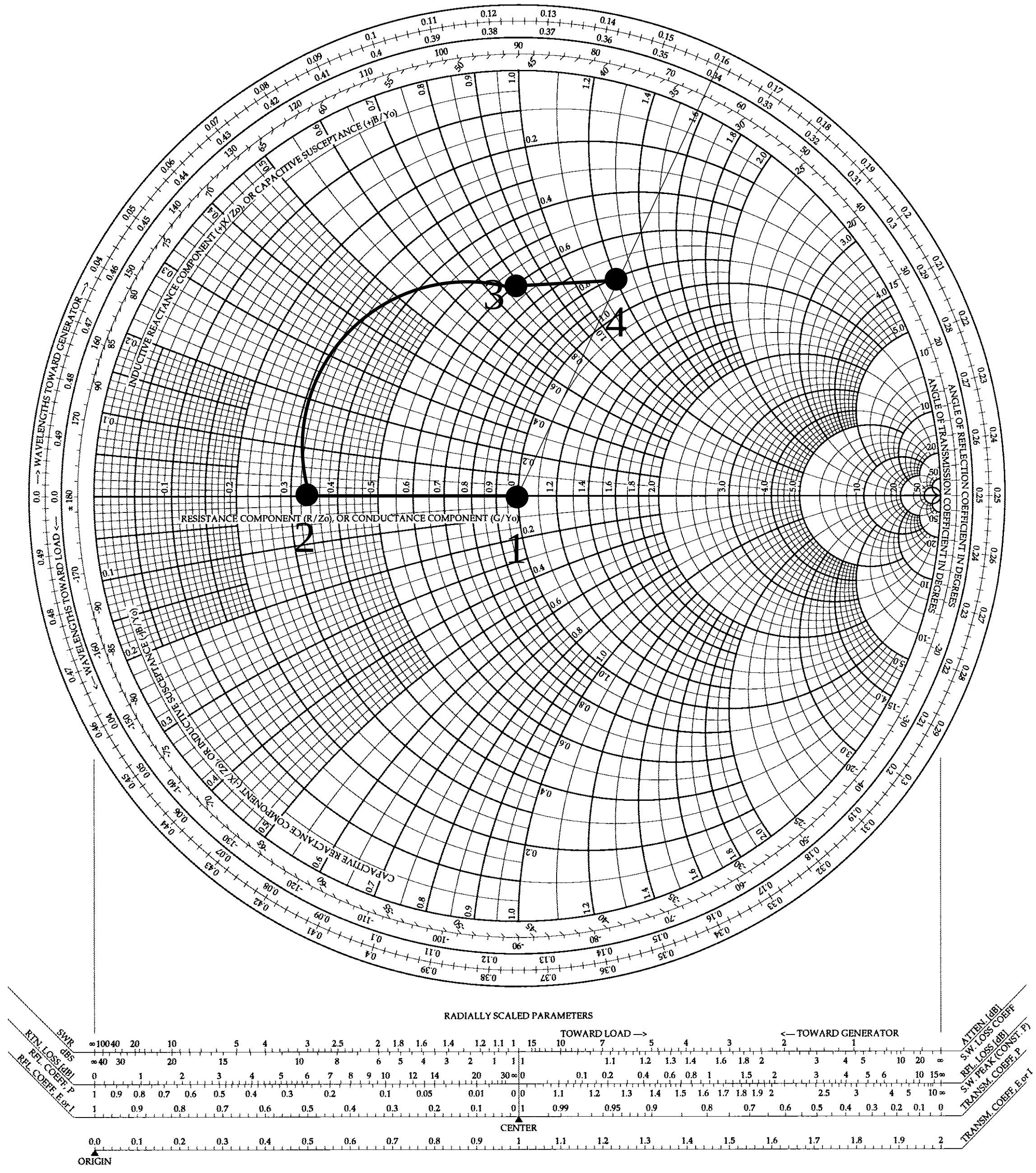


We begin in region 1, where $Z_3 = \eta_3$, so that $z(O_3) = 1$. That is labeled as point 1 on the Smith chart. Next we know that $Z(O_3) = Z(O_2)$, and thus $z(O_2) = \frac{Z(O_3)}{\eta_2} = z(O_3)\frac{\eta_3}{\eta_2} = z(O_3)\frac{1}{3} = \frac{1}{3}$. That is point 2. Next, we rotate by $\lambda/8$ toward the generator to get $z(-d_2)$. That is a 90° clockwise rotation. That gets us to point 3, where we have $z_2(O_2) = 0.6 + j0.8$. Next, we want $z(O_1)$. Analagous to above, we see that $z(O_1) = z(-d_2)\frac{\eta_2}{\eta_1} = (0.6 + j0.8)\frac{3}{2} = 0.9 + j1.2$. That point is marked 4. Finally we read off the angle and magnitude. I get

$$\Gamma_{11} = 0.6e^{j65^\circ}$$

The Complete Smith Chart

Black Magic Design



4. **A wave is incident, in medium 1, onto an interface with medium 2, at the Brewster angle with only a parallel electric field $E_{\parallel} = 1\,\mathrm{V/m}$. Assuming $\mu_1 = \mu_2 = \mu_0$, $\epsilon_2 = 2\epsilon_1$, what is the amplitude of the transmitted electric field? Explain how it is not a violation of the conservation of energy that the transmitted field is different from the incident field eventhough no wave is reflected.**

The Brewster angle is found from

$$\tan^2 \theta_B = \frac{\epsilon_2}{\epsilon_1}$$

$$\theta_B = \tan^{-1} \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \tan^{-1} \sqrt{2} = 54.74°$$

The transmission angle is found from

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

$$\theta_t = \sin^{-1} \left(\frac{\epsilon_1}{\epsilon_2} \sin \theta_i \right) = \sin^{-1} \left(\frac{1}{\sqrt{2}} \sin\ 54.74° \right) = 35.27°$$

The transmission coefficient is

$$\tau_{\|} = \frac{2 \cos \theta_i}{\cos \theta_t + \sqrt{\frac{\epsilon_2}{\epsilon_1}} \cos \theta_i} = \frac{2 \cos\ 54.74°}{\cos\ 35.27° + \sqrt{2} \cos\ 54.74°} = 0.707$$

$$E^t = 0.707\,\mathrm{V/m}$$

It is not a violation of the conservation of energy, because the energy transmitted is the cross-product of the electric and magnetic fields, and the magentic field is correspondingly smaller such that the product comes out to be the same.

The Complete Smith Chart

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