EE 434 Electromagnetic Waves, Spring 2010
Exam 2 April 5, 2010
Solution

Rules: This is a closed-book test. You may use your calculator, the attached formula sheet, and blank note paper. The exam will last 50 minutes. Each problem counts equally toward your grade. If you are short on time, do problem 6 last.

1. **Consider a waveguide filled with Teflon with $\epsilon_r = 2.1$, dimension $a = 2\,\text{cm}$, and $b = 1.5\,\text{cm}$. Which mode has the lowest cutoff-frequency?**

   It is either the TE_{01} or TE_{10} mode. Since we divide by b in the first case and divide by a in the second case, and $a > b$, the smallest cutoff frequency will be that of TE_{10}.

2. **For the above waveguide, if the frequency is twice the cutoff frequency, what is the phase velocity and the group velocity?**

   First compute the factor

   $$f = \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} = \sqrt{1 - \left(\frac{1}{2}\right)^2} = 0.866$$

   Then, the phase velocity is

   $$v_p = \frac{1}{\sqrt{\mu\epsilon_r\epsilon} f} = \frac{c}{\sqrt{\epsilon_r} f} = 2.39 \times 10^8\,\text{m/s}$$

   and the group velocity is

   $$v_g = \frac{1}{\sqrt{\mu\epsilon_r\epsilon}} f = \frac{cf}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8 \times 0.866}{\sqrt{2.1}} = 1.79 \times 10^8\,\text{m/s}$$

3. **A waveguide mode in a air-filled waveguide has impedance $600\,\Omega$. Is it a TE or TM mode? What is the wave group velocity?**

   The impedance of the TE mode is larger than the corresponding plane wave, so this must be a TE mode. We first find the factor

   $$f = \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} = \frac{\sqrt{\frac{\mu}{\epsilon}}}{\eta_{TE}} = \frac{377}{600} = 0.628$$

   The group velocity is then

   $$v_g = fc = 0.628 \times 3 \times 10^8 = 1.88 \times 10^8\,\text{m/s}$$

4. **Sketch the transverse electric field of the TE_{32} mode and the transverse magnetic field of the TM_{23} mode. Label the direction of each field line.**

5. **A air-filled waveguide of dimensions $a = 3\,\text{cm}$ and $b = 3\,\text{cm}$ is excited at 20 GHz. Name all the possible modes that could be excited.**

First find all critical frequencies which are below 20 GHz.

$$f_{c,mn} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

n m	0	1	2	3	4	5
0		5.0	10.0	15.0	20.0	25.0
1	5.0	7.1	11.2	15.8	20.6	
2	10.0	11.2	14.1	18.0	22.3	
3	15.0	15.8	18.0	21.2		
4	20.0					

The valid modes have critical frequency less than 20 GHz. The TE modes may have either m or n zero, so valid TE modes are TE_{10}, TE_{20}, TE_{30}, TE_{01}, TE_{11}, TE_{21}, TE_{31}, TE_{02}, TE_{12}, TE_{22}, TE_{32}, TE_{03}, TE_{13}, TE_{23}.

TM modes have both m and n nonzero, so valid TM modes are TM_{11}, TM_{21}, TM_{31}, TM_{12}, TM_{22}, TM_{32}, TM_{13}, TM_{23}.

6. **A air-filled waveguide has dimensions $a = 3\,\text{cm}$, and $b = 2\,\text{cm}$. A TE_{01} mode of frequency $f = 10\,\text{GHz}$ and axial magnetic field amplitude 1 A/m propagates in the mode. Write expressions for the transverse fields, substituting numerical values.**

The axial magnetic field is found in the equations sheet, as are the expressions for the transverse fields. The axial field is

$$H_z = H_0 \cos\left(\frac{\pi}{b}y\right)\exp\left(j\omega t - j\beta z\right)$$

Then the complex fields are

$$\begin{aligned}
E_x =& H_0 \frac{j\omega\mu\frac{\pi}{b}}{\left(\frac{\pi}{b}\right)^2}\sin\left(\frac{\pi}{b}y\right)\exp\left(j\omega t - j\beta z\right)\\
E_y =& 0\\
H_x =& 0\\
H_y =& H_0 \frac{\gamma\frac{\pi}{b}}{\left(\frac{\pi}{b}\right)^2}\sin\left(\frac{\pi}{b}y\right)\exp\left(j\omega t - j\beta z\right)
\end{aligned}$$

The real fields are then

$$
\begin{aligned}
E_x =& -\frac{\omega\mu b H_0}{\pi}\sin\left(\frac{\pi}{b}y\right)\sin\left(\omega t-\beta z\right)\\
E_y =& 0\\
H_x =& 0\\
H_y =& -\frac{\beta b H_0}{\pi}\sin\left(\frac{\pi}{b}y\right)\sin\left(\omega t-\beta z\right)
\end{aligned}
$$

Next, $\omega = 2\pi\times 10\times 10^9 = 6.28\ \times 10^{10}\,\mathrm{s}^{-1}$,

$$
\begin{aligned}
\omega_{c,01} =& \frac{1}{\sqrt{\mu\epsilon}}\frac{\pi}{b}\\
=& 3\times 10^8\frac{\pi}{0.02} = 4.71\times 10^{10}\,\mathrm{s}
\end{aligned}
$$

$$
\beta = \omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{\omega_{c,01}}{\omega}\right)^2} = \frac{6.28\times 10^{10}}{3\times 10^8}\sqrt{1-\left(\frac{4.71}{6.28}\right)^2} = 138\,\mathrm{m}^{-1}
$$

The coefficient on E_x is then

$$
E_{0x} = -\frac{\omega\mu b H_0}{\pi} = -\frac{6.28\times 10^{10}\times 4\times\pi\times 10^{-7}\times 0.02\times 1}{\pi} = -502\,\mathrm{V/m}
$$

and the coefficient on H_y is

$$
H_{0y} = -\frac{\beta b H_0}{\pi} = -\frac{138\times 0.02\times 1}{\pi} = -0.879\,\mathrm{A/m}
$$