EE 434 Electromagnetic Waves, Spring 2010
Exam 3 May 3, 2010
Solutions

1. **Using $\vec{H}$, compute the distance from a infinitesimal antenna at which the near field and far field are equal in magntiude. Assuming this is approximately correct for an antenna of length $\lambda/2$, within how many antenna lengths does the near field dominate?**

   The near field scales as $\frac{1}{r^2}$, and the far field scales as $\frac{\beta}{r}$. They are equal when

   $$\frac{1}{r^2} = \frac{\beta}{r}$$

   or

   $$r = \frac{1}{\beta}$$

   In terms of antenna lengths for an antenna of length $\lambda/2$, that is

   $$\frac{r}{l} = \frac{2}{\lambda\beta} = \frac{2\lambda}{2\pi\lambda} = \frac{1}{\pi}$$

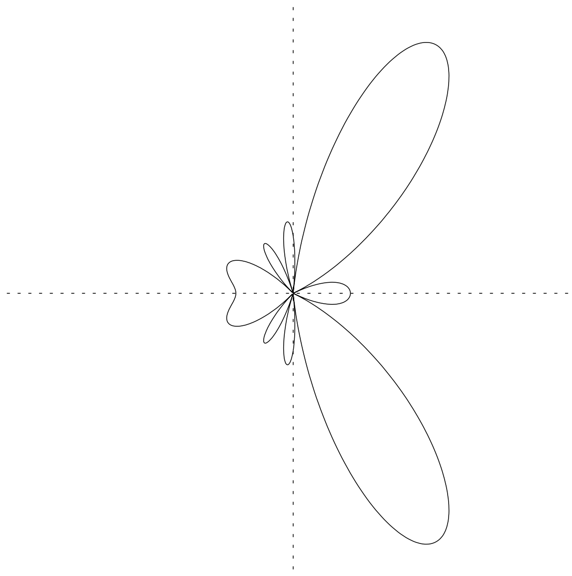
   So the near-field only dominates to about one third antenna length.

2. **Plot the array factor for a equi-spaced array of 5 antennas, spaced $\lambda/2$, and with phase differences $\pi/2$.**

   In this case we have the standard array factor formula,

   $$\text{AF} = \frac{1}{5}\frac{\sin\left(\frac{5}{2}\chi\right)}{\sin\left(\frac{1}{2}\chi\right)} \qquad \chi = \beta d\cos\phi - \psi$$

   where $d = \frac{\lambda}{2}$, $\psi = \frac{\pi}{2}$, and thus $\beta d = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi$. Plot it using the graphical approach. Here is the array factor plotted.



3. **Derive the real (i.e. no complex exponentials) expression for the array factor from an array of four elements with positions and phases $(-\boldsymbol{\lambda}, -\boldsymbol{\pi})$, $(-\boldsymbol{\lambda}/4, -\boldsymbol{\pi}/4)$, $(\boldsymbol{\lambda}/4, \boldsymbol{\pi}/4)$, and $(\boldsymbol{\lambda}, \boldsymbol{\pi})$, using the large-distance approximation.**

   Begin by writing the expression for the total electric field,

$$E = V_0 \sum_{i=1}^{4} \frac{e^{-j(\beta r_i + \psi_i)}}{r_i}$$

   Next we simplify by approximating in the denominator $r_i = r$, where r is the distance to the center of the array, and in the complex exponential we approximate $r = r - \beta d_i \cos\phi$. d_i is the first value in each of the parentheses, and ψ_i is the second value in each of the parentheses. Factoring out $e^{-j\beta r}$, and $\frac{1}{r}$ we can then write

$$E = V_0 \frac{e^{-j\beta r}}{r} \left[e^{j(\beta d_1 \cos\phi - \psi_1)} + e^{j(\beta d_2 \cos\phi - \psi_2)} + e^{j(\beta d_3 \cos\phi - \psi_3)} + e^{j(\beta d_4 \cos\phi_4 - \psi_4)} \right]$$

   The factor in square brackets is the array factor. Note that $d_1 = -d_4$, $\psi_1 = -\psi_4$, etc, we can write

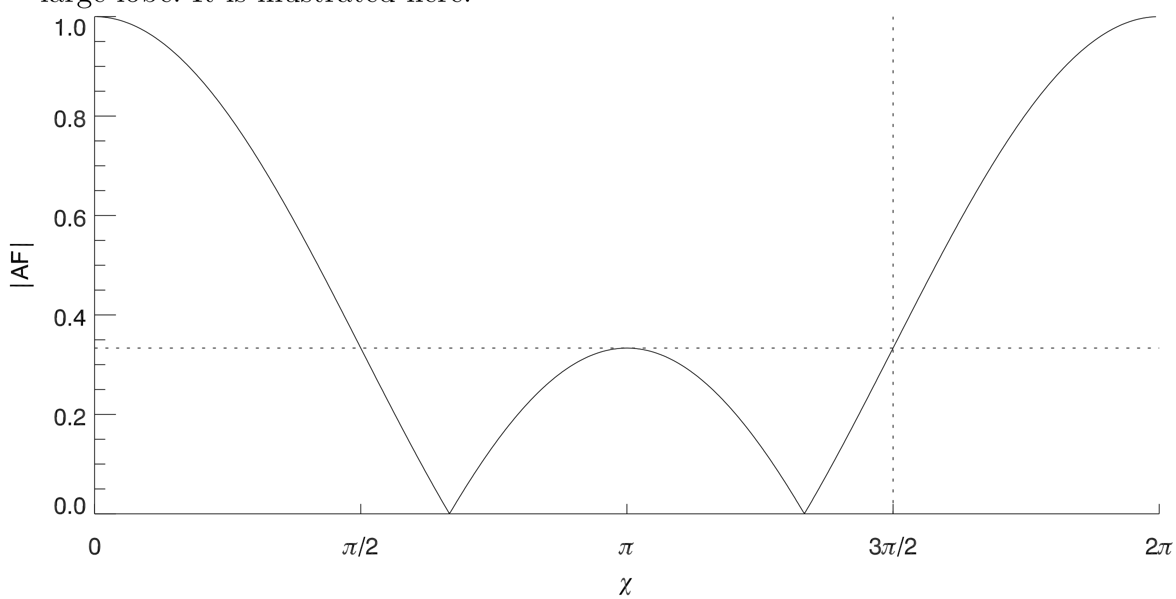
$$\mathrm{AF} = 2 \left[\cos\left(\beta_3 d_3 \cos\phi - \psi_3\right) + \cos\left(\beta_4 d_4 \cos\phi - \psi_4\right) \right]$$

   Inserting $\beta d_3 = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$, and $\beta d_4 = \frac{2\pi}{\lambda}\frac{\lambda}{=}2\pi$, we get

$$\mathrm{AF} = 2 \left[\cos\left(\frac{\pi}{2}\cos\phi - \frac{\pi}{4}\right) + \cos\left(2\pi\cos\phi - \pi\right) \right]$$

4. **Consider an antenna array of 3 elements with equal spacing, $\boldsymbol{d}$, and equal phase difference, $\boldsymbol{\psi}$ (i.e. the kind we considered the most). Choose $\boldsymbol{d}$ and $\boldsymbol{\psi}$ such that there is only a single large lobe and multiple smaller lobes of equal size. The large lobe should form a doughnut in the plane perpendicular to the array axis, and it should be as narrow as possible! You can solve this problem by carefully examining the plot in the equations sheet. Once you have selected $\boldsymbol{d}$ and $\boldsymbol{\psi}$, plot the array factor.**

   First, it is clear that $\psi = 0$ because that will place the large lobe at the center of the circle of radius βd, and thus point it upward and downward as required. Next, find the largest value for βd. It will be such that the edge of the circle is part way up the next large lobe. It is illustrated here.



   It turns out this is for $\chi = \frac{3\pi}{2}$. Next, we use that as the value for βd, and plot the array factor in the usual way. Here it is.