EE 434 Electromagnetic Waves, Spring 2010
Exam 4 May 11, 2010
Solution

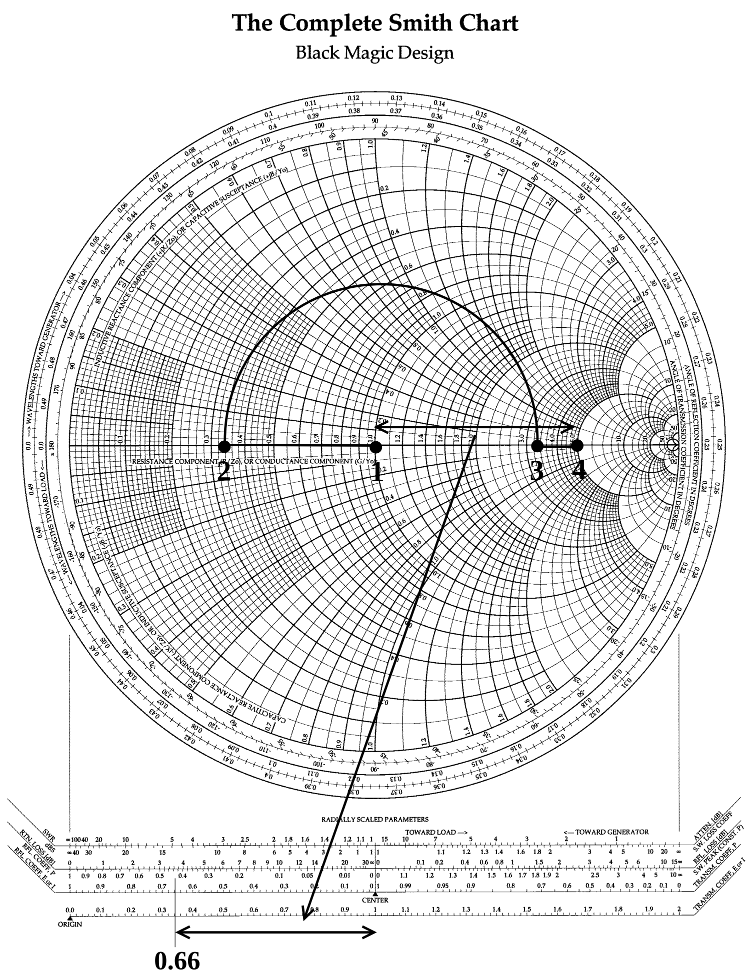
Reflection and transmission

1. **Three adjacent dielectric regions have impedances $\boldsymbol{\eta_1 = 100\,\Omega}$, $\boldsymbol{\eta_2 = 150\,\Omega}$, and $\boldsymbol{\eta_3 = 50\,\Omega}$. Regions 1 and 3 extend to infinity, and region 2, between the other two, has thickness $\boldsymbol{d_2 = \frac{\lambda_2}{4}}$. Use a Smith chart to determine the reflection coefficient in region 1 at the 1-2 interface.**

   In region 3 at $z = 0$, we have $\Gamma_3 = 0$, and thus $Z_3 = \eta_3$, or $z_3 = \frac{Z_3}{\eta_3} = 0$. Thus we begin at the center of the Smith chart, point 1. Next, $Z_3(0) = Z_2(0)$, or

   $$z_2 = \frac{Z_2}{\eta_2} = \frac{Z_3}{\eta_2} = \frac{\eta_3}{\eta_2} z_3 = \frac{50}{150} = \frac{1}{3}$$

   This is point 2. Next, we rotate in the direction of the generator a quarter wavelength to $z = -d_2$. We are now at point 3, where $z_2(-d_2) = 3.3$. Next, $Z_1(0) = Z_2(-d_2)$, or $z_1 = \frac{\eta_2}{\eta_1} z_3 = \frac{150}{100} \times 3.3 = 4.95$. This is point 4. Finally we can read the reflection coefficient,



$$\Gamma_{11} = 0.66$$

2. **Next, remove region 2, placing regions 1 and 3 in contact. Assuming the materials are not magnetic, beyond what incidence angle in material 3 is there no light transmitted to region 1?**

   First we need to find n_1 and n_3. Since $\mu_1 = \mu_3 = \mu_0$, $n = \sqrt{\epsilon_r}$, and we can find ϵ from

$$\eta = \eta_0 \sqrt{\frac{\mu_r}{\epsilon_r}} = \eta_0 \frac{1}{\sqrt{\epsilon_r}}$$

   or

$$n = \sqrt{\epsilon_r} = \frac{\eta_0}{\eta}$$

   Thus, $n_1 = \frac{377}{100} = 3.77$, and $n_3 = \frac{377}{50} = 7.54$. We are asked for the critical angle for total internal reflection in region 3, which is

$$\sin\theta_c = \frac{n_1}{n_3}$$

$$\theta_c = \sin^{-1}\frac{3.77}{7.54} = 30^\circ$$

3. **At what incidence angle, and what orientation of the wave, is there no reflection in region 1?**

   This is the Brewster angle. Waves with electric field in the incidence plane have zero reflection at the Brewster angle. The angle is

$$\tan\theta_B = \sqrt{\frac{\epsilon_3}{\epsilon_1}} = \frac{n_3}{n_1}$$

   The Brewster angle is then

$$\theta_B = \tan^{-1}\frac{n_3}{n_1} = \tan^{-1}\frac{7.54}{3.77} = 63.4^\circ$$

Waveguide

4. **A air-filled waveguide has dimensions $a = 2.5\,\text{cm}$, and $b = 5.0\,\text{cm}$. Name all the possible modes which can propagate at a frequency $f = 8\,\text{GHz}$.**

   The crictical frequency for a mode must be smaller than the excitation frequency for propagation to take place. The critical frequency is

$$f = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$
$$= 3 \times 10^8 \sqrt{\left(\frac{m\pi}{0.025}\right)^2 + \left(\frac{n\pi}{0.05}\right)^2}$$

Next, make a table of the frequencies, in GHz

m / n	0	1	2	3
0		3.0	6.0	9.0
1	6.0	6.7	8.5	
2	12			

From this we can see that the TE_{01}, TE_{02}, TE_{10}, TE_{11}, and TM_{11} modes can propagate in this waveguide at this frequency.

5. **At what speed does a signal propagate through the waveguide in the mode with the largest cutoff frequency?**

   A "signal" propagates at the group velocity. The modes with the largest cutoff frequency are the TE_{11} and TM_{11} modes. The group veloctiy is

$$v_g = c\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} = 3 \times 10^8 \sqrt{1 - \left(\frac{6.7}{8}\right)^2} = 1.64 \times 10^8 \text{ m/s}$$

Antennas

6. **Plot the array factor of an array of 5 antennas, spaced $d = \lambda/4$, and with phase different $\psi = \pi/2$ between adjacent elements.**