6. In a rectangular wave guide, the time-domain expressions for the transverse electric field component \( E_x \) is given by

\[
E_x = A \cos\left(\frac{\pi x}{a}\right) \sin\left(\frac{2\pi y}{b}\right) \sin(7\pi \times 10^{10} t - \beta z)
\]

Determine the following:
(a) Operating mode.
(b) Frequency of operation.
(c) Propagation constant \( \beta \) if the wave-guide dimensions are \( a = 2.3 \) cm and \( b = 1.2 \) cm. Assume the wave guide is filled with air.
(d) Cutoff frequency and wave impedance.

7. The \( x \) component of the magnetic field intensity associated with a TM mode is given by

\[
H_x = 7 \sin\left(\frac{3\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right) \sin(\omega t - \beta z)
\]

(a) Determine the operating wave-guide mode.
(b) If \( a = 2.3 \) cm and \( b = 1.2 \) cm, calculate the cutoff frequency of this mode assuming an air-filled wave guide.
(c) Calculate \( \omega \) so that the operating frequency is 25 percent higher than the cutoff frequency.
(d) Calculate the propagation constant \( \beta \) at the frequency in part (c).

8. An air-filled rectangular wave guide is operating in the TM\(_{21}\) mode at a frequency \( f \) that is 27 percent above the cutoff frequency. The wave-guide dimensions are \( a = 11.5 \) cm and \( b = 6 \) cm.
(a) Calculate the cutoff and operating frequencies.
(b) Calculate the phase constant \( \beta \) and the intrinsic wave impedance.
(c) Write complete expressions (substitute numerical values) for the electric and magnetic fields associated with the mode of operation.

9. The cutoff frequency of the TM\(_{12}\) mode in an air-filled rectangular wave guide is 10 GHz, whereas the cutoff frequency of the TM\(_{21}\) mode is 6 GHz.
(a) Calculate the \( a \) and \( b \) dimensions of this wave guide.
(b) Calculate the cutoff frequency of the TM\(_{11}\) mode.
(c) Calculate the cutoff frequency of the TM\(_{11}\) mode if the wave guide is filled with teflon material of \( \epsilon = 2.1\epsilon_0 \).

10. The cutoff frequency of the TE\(_{10}\) mode is 10 GHz, whereas the cutoff frequency of the TE\(_{03}\) mode is 60 GHz. The wave guide is filled with a glass dielectric of \( \epsilon = 4\epsilon_0 \).
(a) Calculate the \( a \) and \( b \) dimensions of the wave guide.
(b) Calculate the cutoff frequency of the TE\(_{10}\) mode in air.
(c) For an air-filled wave guide of the same dimensions as calculated in part a, calculate the number of possible modes if the excitation frequency is 65 GHz.

11. Design a rectangular wave guide operating in the TE\(_{10}\) mode. For minimum attenuation consideration, select \( a = 2b \). Determine the dimensions such that the middle frequency of the operating band for only the TE\(_{10}\) mode is 9 GHz.

12. Determine the surface currents on the walls of a rectangular wave guide supporting the lowest TM mode. Sketch the variations of these surface currents on the broad top wall \( y = b \) of the wave guide.
13. A rectangular wave guide is operating in the TE\textsubscript{10} mode. The wave-guide dimensions are \(a = 7.62\) cm and \(b = 4\) cm, and the total power transmitted is 1.2 W at an operating frequency of 30 percent higher than the cutoff frequency.
   (a) Calculate the cutoff frequency, operating frequency, and the intrinsic wave impedance, assuming an air-filled guide.
   (b) Determine the amplitude of the electric field associated with the given transmitted power.
   (c) Write complete expressions for the electric and magnetic fields associated with this mode. Substitute numerical values for all the constants.

14. A slotted wave guide of dimensions \(a = 7.6\) cm and \(b = 3.8\) is used to measure the wave guide wavelength and the operating frequency. If the distance between two successive minima is 9 cm, and assuming TE\textsubscript{10} mode of operation in air, calculate the following:
   (a) Wave-guide wavelength \(\lambda_g\), cutoff frequency, and operating frequency.
   (b) Propagation constant and intrinsic impedance.

15. The slotted wave guide of problem 14 is terminated by a horn antenna, and the measured VSWR is 2.1. When the horn antenna is replaced by a short circuit, a 3-cm shift towards the load is observed in the position of the first minimum. Calculate the input impedance of the horn antenna.

16. A rectangular wave guide is loaded with a dielectric material of \(\varepsilon_r = 50 - j12\). The high-dielectric constant \(\varepsilon' = 50\) was desired to reduce the cutoff frequency, whereas the dielectric losses \(\varepsilon'' = 12\) could not be avoided. The wave guide dimensions are \(a = 2.54\) cm and \(b = 1.3\) cm.
   (a) Because \(\varepsilon_r\) is complex in this case, the cutoff frequency in equation 8.36 is expected to be complex. Calculate a real value of the cutoff frequency of the TE\textsubscript{10} mode based on the real part of \(\varepsilon_r\).
   (b) Based on an operating frequency 10 percent higher than the cutoff frequency calculated in part a, calculate the attenuation per unit length as a result of the unavoidable dielectric losses.
   (c) At what distance along the guide is the transmitted power reduced to 50 percent of its initial value?

17. Assume an air-filled rectangular wave-guide made of copper of conductivity \(\sigma = 5.8 \times 10^7\) S/m, with dimensions \(a = 7.6\) cm and \(b = 4\) cm, and a TE\textsubscript{10} mode of operation at a frequency \(f = 1.1f_c\). Calculate the attenuation constant owing to ohmic losses in the wave-guide walls. Also calculate the distance along the axis at which the power is reduced to 95 percent of its initial value. Comment on the importance of the ohmic losses in this case.

18. Assume an X-band air-filled rectangular wave guide of dimensions \(a = 2.3\) cm and \(b = 1.2\) cm. The operating mode is the TE\textsubscript{10} mode. Calculate the maximum power that can be transmitted in the wave guide if the breakdown electric field in air is \(2 \times 10^6\) V/m and a minimum 25 percent safety factor is required. The operating frequency is 30 percent higher than the cutoff frequency for the TE\textsubscript{10} mode.

19. In example 8.3, it was shown that the propagation of various modes in wave guides may be interpreted in terms of a zigzag motion of plane waves bouncing back and forth between the wave-guide walls. As a result, we may identify three types of velocities.
   (a) The velocity of propagation in the medium filling the wave guide. This velocity is denoted by \(v\) in Figure P8.19 and is actually the velocity of propagation of plane waves at an angle \(\theta\). \(v = 1/\sqrt{\mu\epsilon}\).
   (b) The phase velocity, which is the velocity of propagation of constant phase planes as projected along the guide axis. This is
\[ v_p = \frac{v}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \]

(c) The third velocity \( v_g \) is known as the group velocity and is the velocity with which the multiple-reflected waves actually travel along the axis of the waveguide. The group velocity \( v_g \) is, hence, the velocity of the energy propagation down the guide. \( v_g \) is given using equation 8.21 by

\[ v_g = \frac{\partial \omega}{\partial \beta} = v \cos \theta = v \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \]

The group velocity in an air-filled waveguide is, hence, less than the velocity of light, whereas the phase velocity is larger. For a rectangular wavelength of dimensions \( a = 7 \) cm and \( b = 4 \) cm and supporting TE_{10} mode, calculate \( v \), \( v_p \), and \( v_g \) at \( f = 1.3f_c \) and for the following cases:

(i) Air-filled waveguide.

(ii) Glass-filled wave guide \( \varepsilon = 4\varepsilon_o \).

(iii) Repeat the above two calculations for \( f = 1.1f_c \) and \( f = 1.9f_c \).

(iv) Comment on your results regarding the relative values of \( v \), \( v_p \), and \( v_g \) in air, glass, and as a function of frequency.

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![Wave-guide modes](image)

**Figure P8.19** (a) Wave-guide modes may be interpreted as superpositions of plane waves bouncing at an angle \( \theta \). (b) Relationship by \( v \), \( v_p \), and \( v_g \). \( v \) is the velocity of propagation along the direction of the bouncing waves, \( v_p \) is the projected velocity of equiphase planes along the wave-guide axis, and \( v_g \) is the projection of \( v \) along the wave-guide axis.

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20. A rectangular wave guide of dimensions \( a = 5 \) cm and \( b = 2.5 \) cm is operating in the TE_{10} mode at a frequency midway between the fundamental TE_{10} and the next higher-order mode.

(a) Calculate the cutoff and operating frequencies for the two cases of an air-filled wave guide and a glass-filled wave guide of \( \varepsilon' = 4 - j0.1 \). *(Hint: Use the assumption \( \varepsilon' \varepsilon'' \ll 1 \) to calculate a real number for \( f_c \).)*

(b) Calculate the attenuation constant as a result of the dielectric losses in the glass material.

(c) If the wave-guide walls are made of copper of \( \sigma = 5.8 \times 10^7 \) S/m, calculate the attenuation constant as a result of ohmic losses in the wave-guide walls for both air- and glass-filled wave guides.

(d) Compare values of the dielectric and ohmic attenuation constants for the glass-filled wave-guide case.