



Partial Fraction Decomposition

[[Home](#)] [[Up](#)] [[Intermediate College Algebra](#)] [[Precalculus Trigonometry](#)] [[Modern Algebra](#)]
 [[Calculus I](#)] [[Calculus II](#)] [[College Geometry](#)] [[Discrete Mathematics](#)]

Partial fractions: (take a fraction and write this fraction as a sum of two or more similar fractions.)

Example:

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

(The LCD is 6.)

$$\frac{1}{2} \left(\frac{3}{3} \right) + \frac{1}{3} \left(\frac{2}{2} \right) = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

Simplified, $\frac{5}{6}$ can be simplified and written as two separate fractions: $\frac{1}{2}$ & $\frac{1}{3}$.

Steps of Decomposition of Partial Fractions.

1. Divide if improper: If $N(x)/D(x)$ is an improper fraction, divide the denominator into the numerator to obtain:

$$\frac{N(x)}{D(x)} = (\text{polynomial}) + \frac{N_1(x)}{D(x)}$$

and apply Steps 2, 3, and 4 (below) to the proper rational expression $N_1(x)/D(x)$.

2. Factor denominator: Completely factor the denominator into factors of the form

$$(px + q)^m \text{ and } (ax^2 + bx + c)^n$$

where $(ax^2 + bx + c)^n$ is irreducible.

3. Linear factors: For each factor of the form $(px + q)^m$, the partial fraction decomposition must include the following sum of m fractions.

$$\frac{A_1}{(px+q)} + \frac{A_2}{(px+q)^2} + \dots + \frac{A_n}{(px+q)^n}$$

4. Quadratic factors: For each factor of the form $(ax^2+bx+c)^n$, the partial fraction decomposition must include the following sum and n fractions.

$$\frac{B_1x+C_1}{ax^2+bx+c} + \frac{B_2x+C_2}{(ax^2+bx+c)^2} + \dots + \frac{B_nx+C_n}{(ax^2+bx+c)^n}$$

Example:

$$\frac{x+7}{x^2-x-6}$$

SOLUTION: Write partial fraction decomposition.

Step 1: Factor denominator.

$$\frac{x+7}{x^2-x-6} = \frac{x+7}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}$$

Step 2: Now find A and B by multiplying both sides by the LCM. In this case the LCM is $(x+2)$ and $(x-3)$.

$$\begin{aligned} (x+2)(x-3) \left[\frac{x+7}{(x+2)(x-3)} \right] &= (x+2)(x-3) \\ \cancel{(x+2)} \cancel{(x-3)} \left[\frac{x+7}{\cancel{(x+2)} \cancel{(x-3)}} \right] &= (x+2)(x-3) \left[\frac{A}{x+2} + \frac{B}{x-3} \right] \\ x+7 &= \frac{(x+2)(x-3)A}{x+2} + \frac{(x+2)(x-3)B}{x-3} \\ x+7 &= A(x-3) + B(x+2) \\ x+7 &= Ax - 3A + Bx + 2B \end{aligned}$$

Step 3: Equate undetermined coefficients.

$$1 = A + B \quad (\text{first equation})$$

Step 4: Equate coefficients of constant terms:

$$7 = -3A + 2B \quad (\text{second equation})$$

Step 5: Solve algebraically with method "2 equations and 2 unknowns". This is done by substitution, graphing, or the addition method.

$$\begin{array}{r} +3 \left\{ \begin{array}{l} A + B = 1 \\ 3A + 2B = 7 \end{array} \right\} \\ \hline 3A + 3B = 3 \\ -3A + 2B = 7 \\ \hline 0 + 5B = 10 \\ B = 2 \end{array}$$

Now substitute equation 2 into equation 1:

$$A + B = 1$$

$$A + 2 = 1 - 2$$

$$A = -1$$

Thus, the solution is:

$$\left[\frac{x+7}{(x+2)(x-3)} \right] = \left[\frac{A}{(x+2)} + \frac{B}{(x-3)} \right] = \left[\frac{x+7}{(x+2)(x-3)} \right] = \left[\frac{-1}{(x+2)} + \frac{2}{(x-3)} \right]$$

Now check:

$$\left[\frac{-1}{(x+2)} + \frac{2}{(x-3)} \right]$$

$$\text{LCM} = (x+2) \text{ and } (x-3)$$

$$\frac{-1}{x+2} \frac{(x-3)}{(x-3)} + \frac{2}{(x-3)} \frac{(x+2)}{(x+2)}$$

$$\frac{-x+3+2x+4}{(x-3)(x+2)} = \frac{(x-7)}{x^2-x-6}$$

$$\frac{(x+7)}{x^2-x-6} \text{ is the original problem!}$$



I strive to provide accurate and error-free documentation concerning all aspects of mathematics. As a student, I do not hold the expertise or experience that my professors do. These notes represent countless hours of study, but they do NOT represent textbook-level proofing and editing. Since this project is managed by one student, I must rely on my peers for assistance. Please report any errors to errors@mathematicshelpcentral.com.

Want to contribute to this site's efforts?

[Become a sponsor!](#) [View Participating Sponsors!](#)

Free Graph Paper

Print Free Graph Paper pdfoad.com/graphpaper/

Ads by Google

Advertise on this site