EE 521 Measurement and Instrumentation

Fall 2006 - Dr. Anders M. Jorgensen

Heterodyne Laser Metrology System

In this project you will use a laser metrology system to measure the position of a piezo-electric positioning device which, nominally, is tracing out a triangular waveform at a nominal frequency of 500 Hz and a nominal amplitude of a few micrometers.

Background: The Navy Prototype Optical Interferometer (NPOI) is located in Flagstaff, AZ. It measures approximately 400 m in diameter, which allows it to obtain a angular resolution in the sky at visible wavelengths ($\sim 500\,\mathrm{nm}$) of approximately

$$\frac{\lambda}{D} = \frac{500\,\mathrm{nm}}{400\,\mathrm{m}} = 10^{-9}\,\mathrm{radians} = 0.3\,\mathrm{mas}$$

One milliarcsecond (mas) is approximately $3 \times 10^{-7\circ}$. By comparison, the resolution of the human eye is approximately 3 arcminutes, or 0.05°. The extremely high resolution of optical interferometers is necessary because stars have very small angular diameters. An interferometer works by collecting light at telescope pairs, transmitting it to a central location and interfering the two beams. The intensity of the combined beam depends on the relative phase of the two beams, and varies periodically with phase difference, completing a full sinusoid as the phase is changed by 2π, or equivalently as the relative paths are changed by one wavelength.

We are really interested in the amplitude of this periodic signal, so we delay one beam in a periodic pattern which when plotted as a function of time looks like a triangular pattern. When one of the beams is delayed in this pattern, the intensity of the combined beam will vary sinusoidally with time at the detector. At the NPOI the amplitude of this triangular variation is a few microns and its period 2 ms. The motion is created by a stack of piezo-electric elements which are driven by a triangular high-voltage wave.

In order to verify that the piezo really does trace out the triangular pattern that we expect we use a two-frequency heterodyne interferometer. The setup at NPOI is very similar to that described in section 7.3.3.2.3 and Figure 7.52 of Northrop (pages 419-421).

Assignment: Use the given metrology data to compute the position of a piezo-electric stack as a function of time. Read Northrop section 7.3.3.2. Each of the frequencies at ω_1 and ω_2 are waves that propagate according to

$$E_1 = E_{1o} \cos(\omega_1 t - k_1 x) \text{ and } E_2 = E_{1o} \cos(\omega_2 t - k_2 x)$$

At the NPOI the two frequencies are produced from a HeNe laser by passing the laser beam through an acousto-optical modulator (AOM). The two frequencies emerging from the AOM are separated by 2 MHz.

1. **Derive expressions for the voltages produces by PD1, $R(t)$, and PD2, $M(t)$.** Note that the photodiodes act as low-pass filters and smooth out the high frequency components above 100 MHz. Simplify the expressions by replacing constant terms with a constant.

2. **Show that the reference signal has constant period.** Write a program which computes the times of positive zero-crossings in the reference signal. Then compute the mean and standard deviation of the time between successive positive zero-crossings.

   Notice that the signals have been high-pass filtered, eliminating the DC component. This means that we conveniently use the zero-crossings as time markers (We could choose any other DC level as our reference level for crossings if we wanted to. However, a level near the middle of the amplitude range gives the best SNR).

   The signal is very stable so that the variability is due to the digitization and discrete time measurements. The width of this peak determines the accuracy with which you can determine zero-crossings.

3. **Create a list of positive zero-crossings in the measuring signal.** Use the same procedure that you used to find the positive zero-crossings in the reference signal. You may want to save this data set as a file.

4. **Compute the average speed of RR1.** The average time between zero-crossings in the measuring signal can be used to measure the average speed of RR1.

5. **Derive position determination algorithm.** The following is the algorithm for converting the metrology signal into position. **Please derive this algorithm.**

   "We maintain a counter `c`. We scan forward in time. Each time we encounter a positive zero-crossing in the reference signal we increment `c`. Each time we encounter a positive zero-crossing in the measuring signal we decrement `c`. `c` $\times \lambda$ gives us a rough measure of the position of RR1, to within a wavelength. But we can do better. In addition, we compute at each zero-crossing in the measuring signal the quantity `d`, which is the time since the last zero-crossing in the reference, divided by the period of the reference. We can now get a more accurate measure of the position, which is (`c+d`) $\times \lambda$."

6. **Compute (and plot for some selected interval(s)) position as a function of time.** With the above algorithm you can now compute the position of RR1 at each time, t_M, where there is a zero-crossing in the measuring signal. As a practical matter I recommend counting the number of measuring signal crossings since the beginning of the file, m, and then estimating the number of reference crossings since the beginning of the file, n, as the **integer** division

   $$n = \frac{t}{\Delta t_{\mathrm{R}}}$$

   The estimated time of the last reference crossing is then

   $$t_R = n \Delta t_R$$

   and the position of RR1, Δx, can then be computed as

   $$\Delta x = (n - m + d) \times \lambda$$

   where $d = \frac{t - t_R}{\Delta t_R}$

   Given the precision with which you can determine zero crossings, what is the precision of the position measurement in nanometers?

7. **BONUS QUESTION FOR EXTRA CREDIT: Plot bode diagram for transfer function.** Estimate and plot the bode diagram for the transfer function from voltage across the piezo stack to position of the piezo, assuming that the system is linear and the voltage is a perfect triangle.

Data: You are given two data files. The first file `reference` samples the output of the reference signal photo diode (PD1 in Northrop Figure 7.52), and the second file `measuring` contains the output of the measuring signal photo diode (PD2). Each signal is sampled at 50 MHz, and they are synchronized such that the i'th measurement in the reference file is measured at the same time as the i'th measurement in the measuring signal. The data values in the file are signed 16-bit (2-byte) integers, and trace out periodic curves. Note that the signals have been high-pass filtered to remove a DC component, and are thus centered around zero instead of around a mean brightness level.

Hints:

1. **Finding positive zero-crossings.** We have a signal $\{S_i\}$. A positive zero-crossing occurs when the $S_i < 0$ and $S_{i+1} \geq 0$. If measurements i and $i+1$ are obtained at times t_i and t_{i+1} then the precise time of the zero-crossing, t_z, can be interpolated using the following relationship

$$\frac{S_i}{t_z - t_i} = \frac{S_{i+1} - S_i}{t_{i+1} - t_i}$$

Be sure to have a the $\geq$ in your algorithm, and not just $<$ and $>$, otherwise the program will skip zero-crossings where a measurement is exactly zero, and give the wrong result.

Reference:

1. A. M. Jorgensen, D. Mozurkewich, J. Murphy, M. Sapantaie, J. T. Armstrong, G. C. Gilbreath, R. Hindsley, T. A. Pauls, H. Schmitt, D. J. Hutter, "Characterization of the NPOI fringe scanning stroke," Proc. SPIE Astronomical Telescopes and Instrumentation, Orlando, FL, SPIE 6268 Advances in Stellar Interferometry, 2006.