Solutions to Homework #1

Problem 1.3

$$\frac{V_o(s)}{P(s)} = \frac{-10s}{s100+1}$$

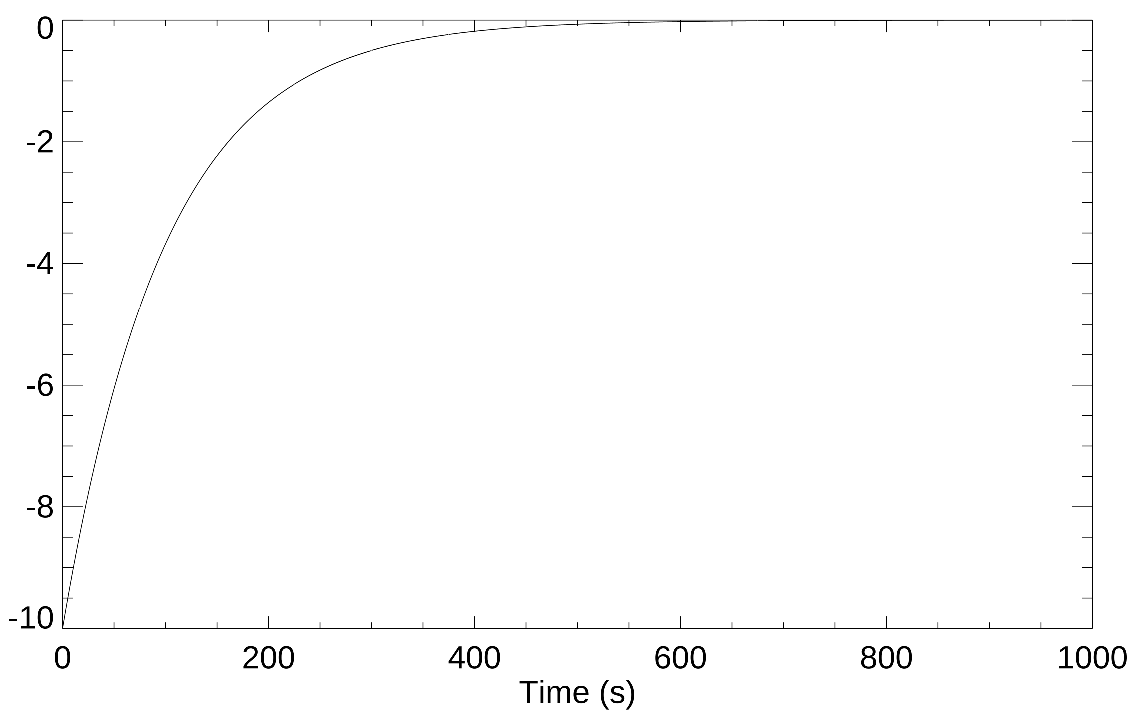
We first want to find the time-evolution, $v_o(t)$. $p(t) = 100u(t)$, so that $P(s) = \frac{100}{s}$. Thus, we get

$$V_o(s) = \frac{-10s}{s100+1}\frac{100}{s} = \frac{-10}{s+\frac{1}{100}}$$

We inverse Laplace transform to get

$$v_o(t) = -10e^{-\frac{t}{100}}$$

(A)



(B)
Peak value is $v_o(t) = -10$V
    99.5% of peak value:

$$v_o(t) = 0.995 \times v_o(0) - 9.95 = -10 \times e^{-\frac{t}{100}}$$

$$t = -100 \times \log(0.995) = 0.501$$

Problem 1.4

$$\frac{I_o(s)}{T(s)} = \frac{1.5 \times 10^{-2}}{(s+0.3)\,(s+0.05)} \frac{\mu\text{A}}{\text{K}}$$

(A)
The input, T, is a step-function in temperature with amplitude $35°C$, $t(t) = 35 \times u(t)$. Then $T(s) = \frac{35}{s}$. We just need to remember to add $20°C$ to the final time-series. We now have

$$I_o(s) = \frac{35 \times 1.5 \times 10^{-2}}{s\,(s+0.3)\,(s+0.05)} \mu\text{A}$$

We need to inverse Laplace transform this expression. The easiest way is probably to perform a partial fractional expansion,

$$\begin{aligned}
I_o(s) =& \frac{35 \times 1.5 \times 10^{-2}}{s\,(s+0.3)\,(s+0.05)} \mu\text{A} \\
=& \frac{A}{s} + \frac{B}{s+0.3} + \frac{C}{s+0.05} \\
=& \frac{A(s+0.3)(s+0.05) + Bs(s+0.05) + Cs(s+0.3)}{s\,(s+0.3)\,(s+0.05)} \\
=& \frac{s^2A + 0.35sA + 0.015A + s^2B + 0.05sB + s^2C + 0.3sC}{s\,(s+0.3)\,(s+0.05)} \\
=& \frac{s^2(A+B+C) + s(0.35A + 0.05B + 0.3C) + 0.015A}{s\,(s+0.3)\,(s+0.05)}
\end{aligned}$$

We can now write

$$\begin{aligned}
A + B + C &= 0 \\
0.35A + 0.05B + 0.3C &= 0 \\
0.015A &= 35 \times 1.5 \times 10^{-2}
\end{aligned}$$

We can solve these to get $A = 35$, $B = 7$, and $C = -42$, so that we have

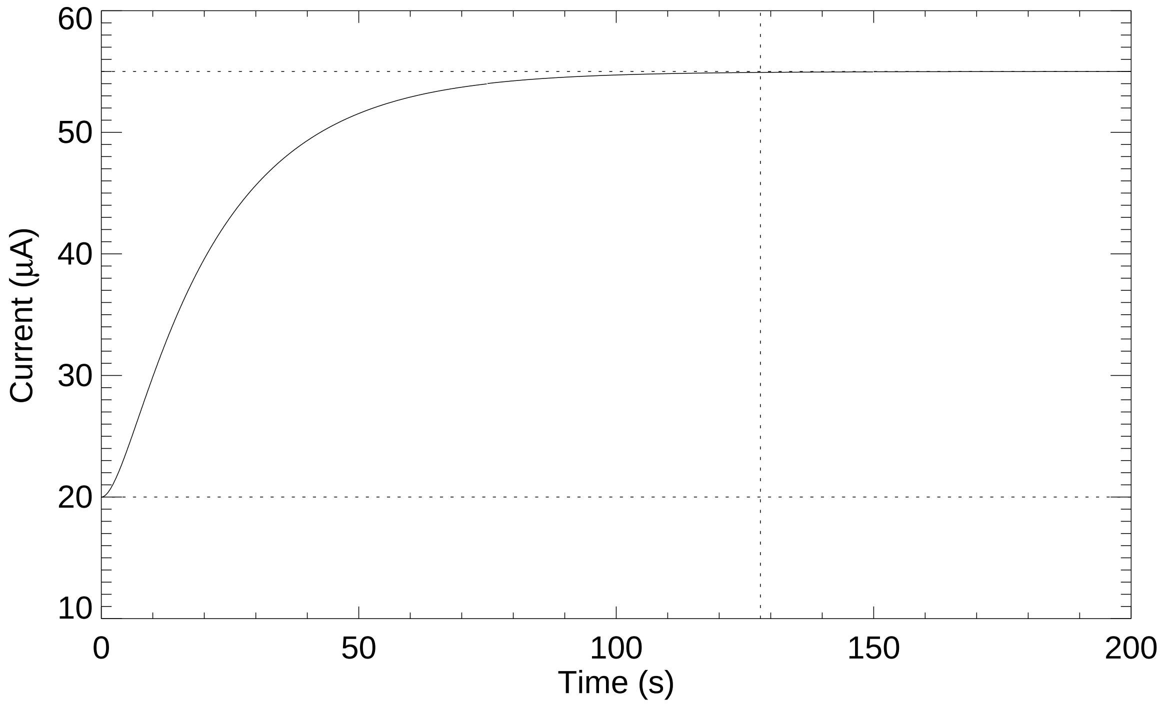
$$I_o(s) = \frac{35}{s} + \frac{7}{s+0.3} - \frac{42}{s+0.05}$$

We can inverse Laplace transform this expression to get

$$i_o(t) = 35 + 7e^{-0.3t} - 42e^{-0.05t}$$

Because we started with $i_o(0) = 20$, the correct solution is

$$i_o(t) = 20 + 35 + 7e^{-0.3t} - 42e^{-0.05t}$$



(B)

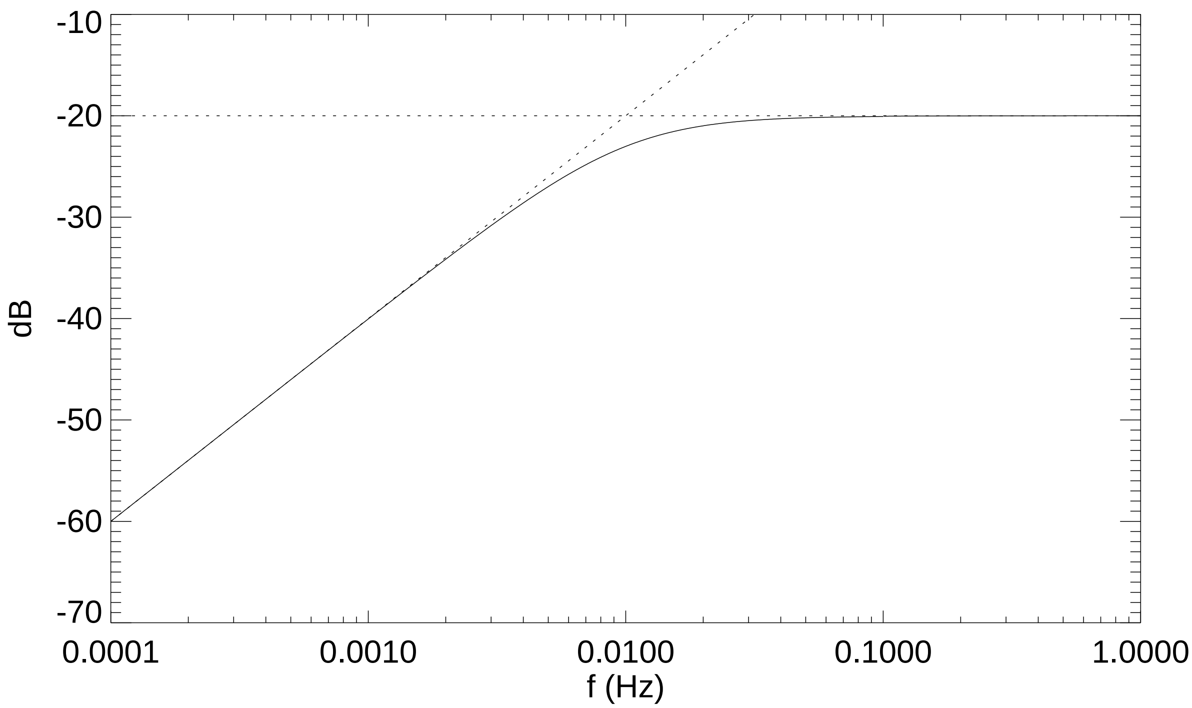
t=119 s

Problem 1.5
We want to plot

$$dB = 20\log_{10}|\frac{V_o}{P}(jf)|$$

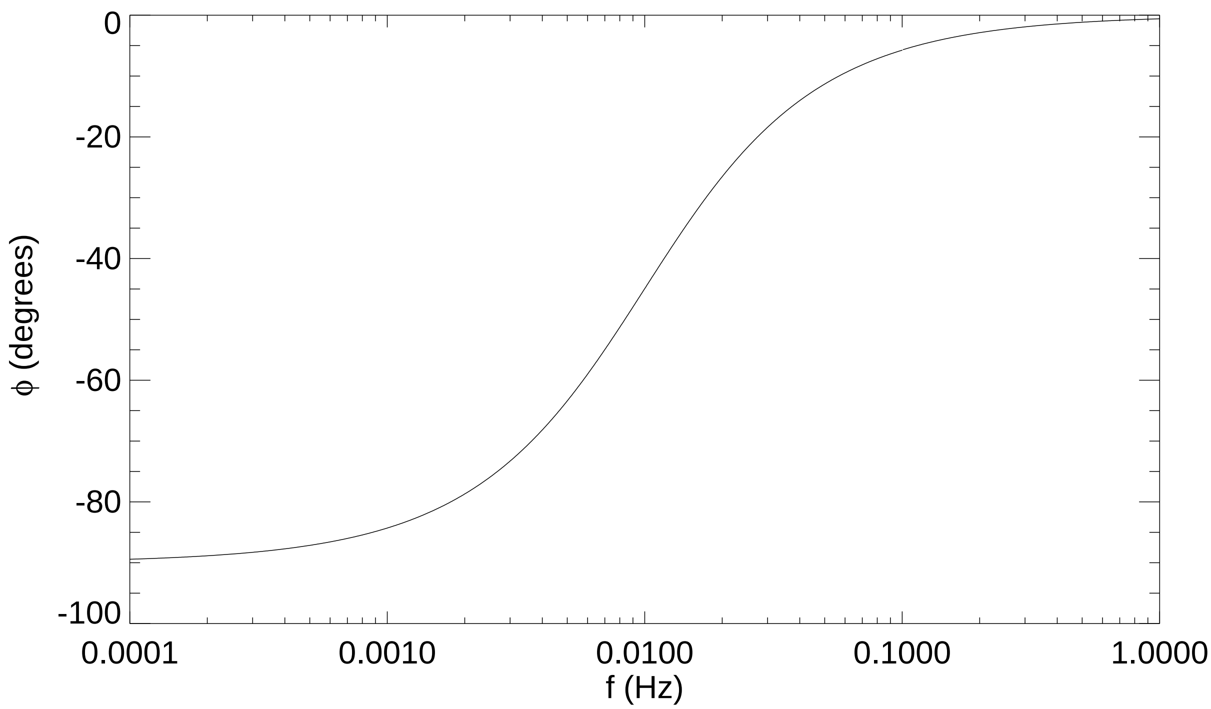
as a function of f. Let's insert $s = jf$,

$$|\frac{V_o}{P}(jf)| = |\frac{-10jf}{100jf+1}| = |\frac{-10jf(-100jf+1)}{(100jf+1)(-100jf+1)}|$$
$$= |\frac{10^3 f^2 - 10jf}{10^4 f^2 + 1}| = \frac{\sqrt{(10^3 f^2)^2 + (10f)^2}}{10^4 f^2 + 1}$$



We were not explicitely asked for the phase, but we can find it as

$$\phi = \tan^{-1}\left(\frac{\text{Imaginary part}}{\text{Real part}}\right) = \tan^{-1}\left(\frac{-10f}{10^3 f^2}\right)$$


0
-20
-40
-60
-80
-100
ϕ (degrees)
0.0001
0.0010
0.0100
0.1000
1.0000
f (Hz)

Problem 1.6
The voltage across a Josephson Junction is

$$E_J = \frac{nf}{K_J} \quad \text{(in V)}$$

We have N of them, so we get

$$V_o = N\frac{nf}{K_J}$$

The input quantities are $N = 550$, $n = 97$, $f = 9.0646279$, $K_J = 483597.9$, so we get

$$V_o = 550 \times \frac{97 \times 9.0646279}{483597.9}\text{V} = 1.00000\text{V} \quad \text{(to 6 significant figures)}$$

Problem 1.7
(A)

$$V_{\text{QHR}} = NV_{\text{JJ}}$$

Since $V_{\text{QHR}} = R_H I_X$, we re-write as

$$\begin{aligned} I_X =& \frac{NV_{\text{JJ}}}{R_H} \\ =& \frac{N\frac{nf_0}{2q/h}}{\frac{h}{q^2k}} \\ =& \frac{Nnf_0q^2k}{2q} \\ =& \frac{1}{2}Nnkf_0q \end{aligned}$$

(B)
Rearrange solution to question A to get

$$N = \frac{2I_X}{nkf_0q}$$

Use $I_x = 150 \times 10^{-6}$, $n = 1$, $k = 7$, $f_0 = 13.374664 \times 10^9$, $q = 1.60217733 \times 10^{-19}$ to get

$$N = 20000.$$

(C)

$$V_{\text{QH}} = I_X\frac{h}{q^2k} = 150 \times 10^{-6}\frac{6.6260755 \times 10^{-34}}{(1.60217733 \times 10^{-19})^2 7} = 0.552687\text{V} \quad \text{(to 6 figures)}$$

or

$$
\begin{aligned}
V_{\mathrm{JJ}} =& N\frac{nf_0}{2q/h} = 20\times 10^3 \frac{1\times 13.374664\times 10^9}{2\times 1.60217733\times 10^{-19}/6.6260755\times 10^{-34}} \\
=& 0.552688\mathrm{V} \quad \text{(to 6 figures)}
\end{aligned}
$$