EE 521 Instrumentation and Measurements
Fall 2007
Solutions for homework assignment #1

Northrop 1.2
The transfer function for an accelerometer is

$$\frac{V_o(s)}{\ddot{X}(s)} = \frac{K_A}{(\tau s+1)^2}$$

where $\tau = 1\,\mathrm{s}$, and $K_A = 0.001\,\mathrm{V/(m/s^2)}$. It is subjected to a step input of acceleration at $t = 0$. Thus, $\ddot{X} = \frac{1}{s}$, and

$$V_o(s) = \frac{K_A}{s(\tau s+1)^2}$$

Doing a partial fraction expansion we get

$$\begin{aligned} V_o(s) =& \frac{A}{s} + \frac{B}{\tau s+1} + \frac{C}{(\tau s+1)^2} \\ =& \frac{A(\tau s+1)^2 + Bs(\tau s+1) + Cs}{s(\tau s+1)^2} \\ =& \frac{A\tau^2 s^2 + A + 2A\tau s + B\tau s^2 + Bs + Cs}{s(\tau s+1)^2} \\ =& \frac{s^2(A\tau^2 + B\tau) + s(2A\tau + B + C) + A}{s(\tau s+1)^2} \end{aligned}$$

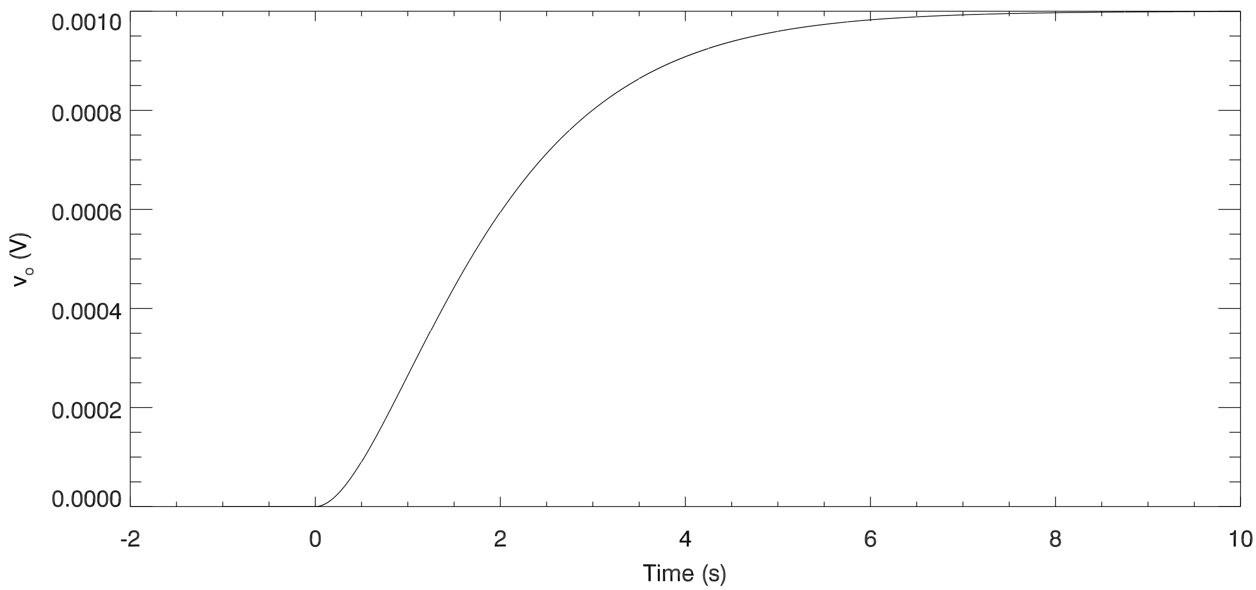
From this we find that $A = K_A$, $A\tau^2 + B\tau = 0$, and $2A\tau + B + C = 0$, so $B = -A\tau = -K_A\tau$, and $C = -2A\tau - B = -2K_A\tau + K_A\tau = -K_A\tau$. Thus

$$\begin{aligned} V_o(s) =& K_A\left[\frac{1}{s} - \frac{\tau}{\tau s+1} - \frac{\tau}{(\tau s+1)^2}\right] \\ =& K_A\left[\frac{1}{s} - \frac{1}{s+\frac{1}{\tau}} - \frac{\frac{1}{\tau}}{\left(s+\frac{1}{\tau}\right)^2}\right] \end{aligned}$$

We inverse Laplace transform each term,

$$v_o(t) = K_A\left[1 - e^{-\frac{t}{\tau}} - \frac{t}{\tau}e^{-\frac{t}{\tau}}\right]u(t)$$

The function looks like this



(a) Time to reach 95% of peak value is 4.7439 s.
(b) Time to reach 99% of peak value is 6.6384 s.
(c) Time to reach 99.9% of peak value is 9.2334 s.
Northrop 1.3
The transfer function of a pressure sensor is

$$\frac{V_o(s)}{P(s)} = \frac{-10s}{100s+1}$$

We apply a pressure step to the sensors,

$$p(t) = 100u(t) \qquad P(s) = \frac{100}{s}$$

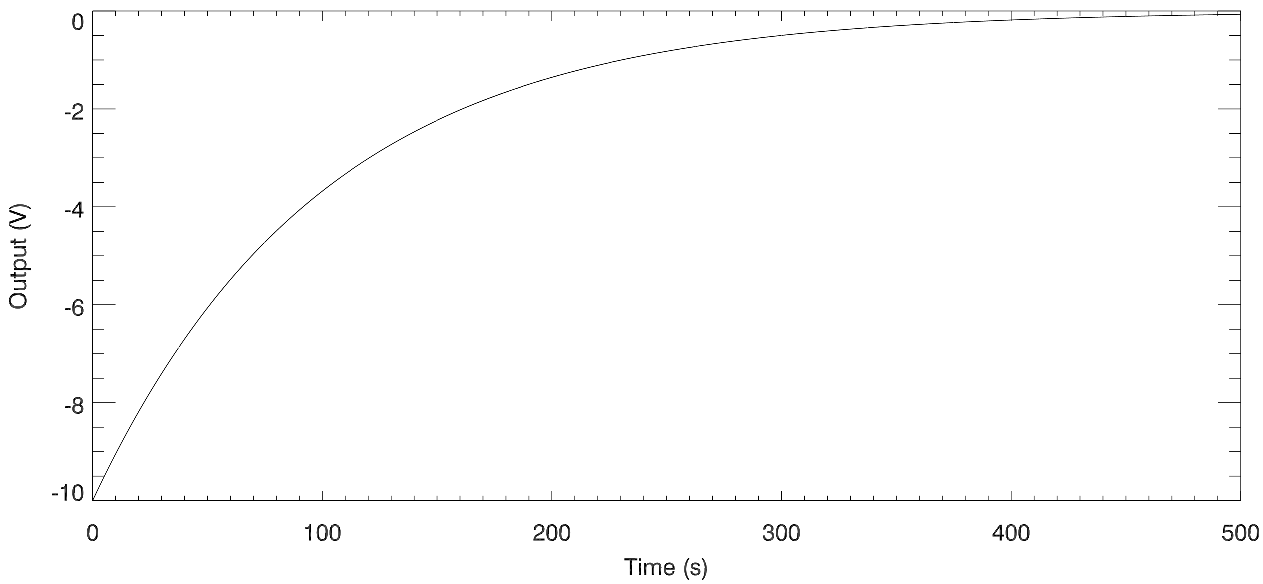
which creates the response

$$V_o(s) = \frac{-10s}{100s+1}\frac{100}{s} = \frac{-10}{s+\frac{1}{100}}$$

The inverse Laplace transform is

$$v_o(t) = -10e^{-\frac{t}{100}}u(t)$$

The problem does not provide much information about the unit, so I will assume it is Volts.
(a) The function looks like this,



(b) The peak value is $v_o(0) = -10\,\mathrm{V}$.
(c) The time to drop to 99.5% of it's peak value is

$$0.995 = \exp\left(-\frac{t_{.995}}{100}\right)$$

$$t_{.995} = -100 \times \log 0.995 = 0.501\,\mathrm{s}$$

Northrop 1.4
The transfer function is

$$\frac{I_o(s)}{T(s)} = \frac{1.5 \times 10^{-2}}{(s+0.3)\,(s+0.05)}\frac{\mu\mathrm{A}}{\mathrm{K}}$$

The sensor is subjected to a temperature step that looks like

$$t(t) = 35u(t)\,\mathrm{K} \qquad T(s) = \frac{35}{s}\,\mathrm{K}$$

So we get

$$I_o(s) = \frac{0.525}{s\,(s+0.3)\,(s+0.05)}\frac{\mu\mathrm{A}}{\mathrm{K}}$$

We do a partial fraction expansion,

$$\begin{aligned}\frac{1}{s(s+0.3)(s+0.05)} =& \frac{A}{s} + \frac{B}{s+0.3} + \frac{C}{s+0.05}\\ =& \frac{A(s+0.3)(s+0.05) + Bs(s+0.05) + Cs(s+0.3)}{s(s+0.3)(s+0.05)}\\ =& \frac{As^2 + 0.35As + 0.015A + Bs^2 + 0.05Bs + Cs^2 + 0.3Cs}{s(s+0.3)(s+0.05)}\\ =& \frac{s^2\,(A+B+C) + s(0.35A + 0.05B + 0.3C) + 0.015A}{s(s+0.3)(s+0.05)}\end{aligned}$$

Thus $0.015A = 1$, $A + B + C = 0$, and $0.35A + 0.05B + 0.3C = 0$, So $A = \frac{200}{3}$, $B = \frac{40}{3}$, and $C = -\frac{240}{3}$. Thus

$$I_o(s) = 0.175 \left[\frac{200}{s} + \frac{40}{s + 0.3} - \frac{240}{s + 0.05} \right]$$

Inverse Laplace transforming we get

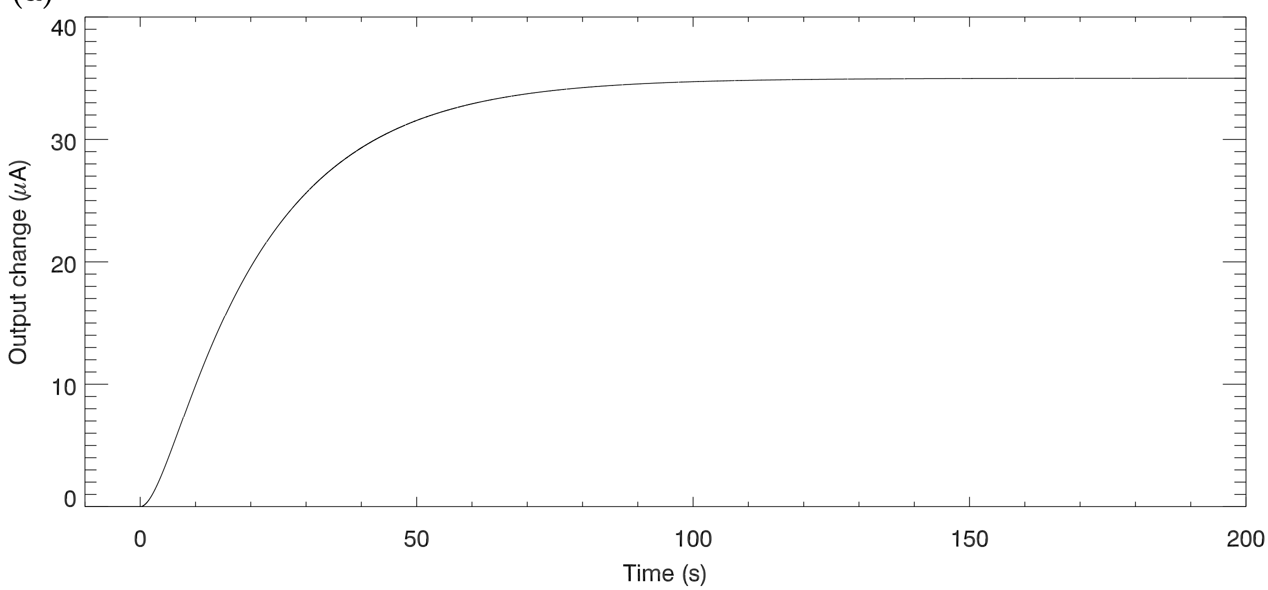
$$i_o(t) = 0.175 \left[200 + 40e^{-0.3t} - 240e^{-0.05t} \right] u(t)\ \mu\text{A}$$

The sensor has a DC response. If the Kelvin scale is to be taken seriously, then the DC response is

$$I_o(0) = \frac{1.5 \times 10^{-2}}{0.015} \frac{\mu\text{A}}{K}$$

Thus before $t = 0$, $i_o(t < 0) = (20 + 273) \times \frac{1.5\times10^{-2}}{0.015}\ \mu\text{A} = 293.0\,\mu\text{A}$. I will ignore this offset and plot the current change here

(a)



(b) The time it will take for $i_o(t)$ to reach 99.8% of it's steady state value of $3.5\,\mu$A is (obtained numerically)

$$t_{.998} = 127.939\,\text{s}$$

Northrop 1.5

We are simply asked to make a bode plot of the transfer function in Problem 1.3,

$$\frac{V_o(s)}{P(s)} = \frac{-10s}{s100 + 1} = H(s)$$

First, let's insert $s = j\omega$, and re-write the transfer function to place the imaginary components in the numerator.

$$
\begin{aligned}
H(j\omega) =& \frac{-10j\omega}{100j\omega + 1} \\
=& \frac{-10j\omega(1 - 100j\omega)}{(1 + 100j\omega)(1 - 100j\omega)} \\
=& \frac{-10j\omega - 1000\omega^2}{1 + 10^4\omega^2}
\end{aligned}
$$

The amplitude is then

$$
|H(j\omega)| = \frac{\sqrt{10^2\omega^2 + 10^6\omega^4}}{1 + 10^4\omega^2}
$$

and the phase is

$$
\angle H(j\omega) = \tan^{-1}\frac{-10\omega}{-1000\omega^2} = \tan^{-1} = \tan^{-1}\frac{1}{100\omega}
$$

The two functions are plotted here