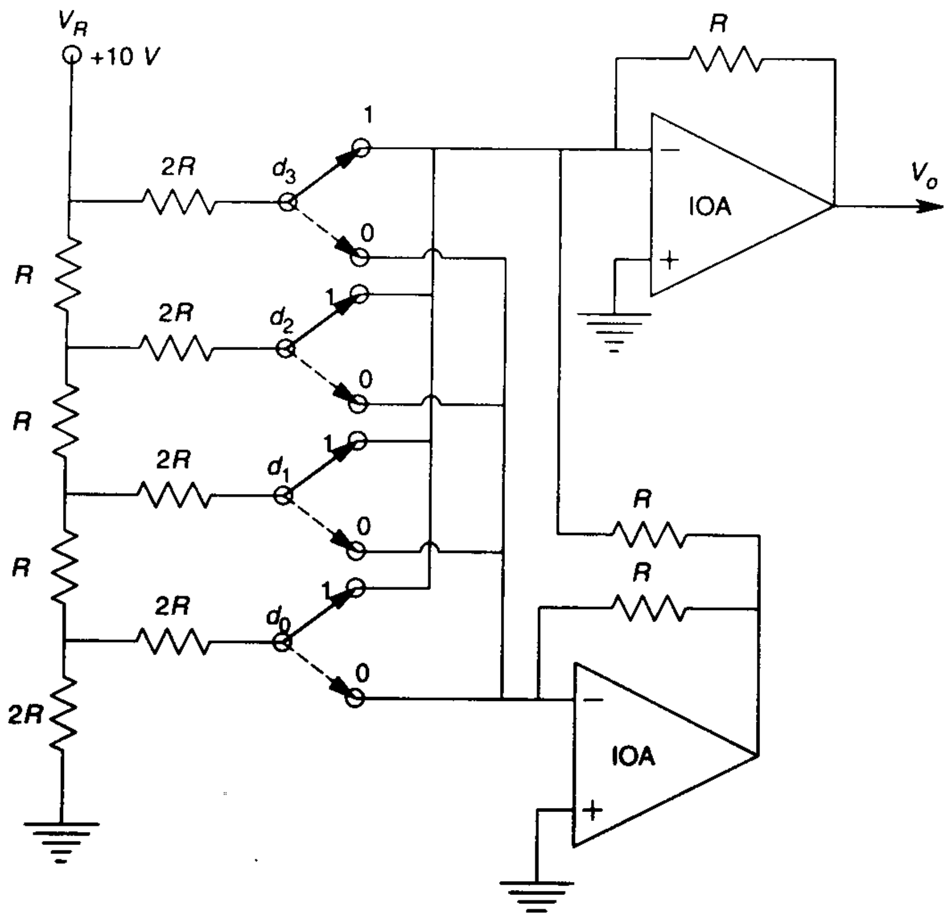
EE 521 Instrumentation and Measurements
Fall 2007
Solutions for homework assignment #6

9.2



Calculate the transfer function, $V_o(d_0, d_1, d_2, d_3)$. First I will calculate the current flowing to each of the two op-amp inverting inputs. Notice that the op-amp inputs are at ground, so regardless of the settings of the switches, the currents always flow to ground through the R-2R resistor network. Thus the points d_i are all at ground. As we have seen in chapter 3, the total current flowing from the point V_R is

$$I_R = \frac{V_R}{R}$$

because the resistance to ground from any point on the vertical wire below V_R is R. We also saw that at each node point half the current flows through the switch d_i, and half flows further down the ladder. Thus, the current through a switch, d_i, I_i, is

$$I_i = \frac{I_R}{2^{4-i}}$$

so that $I_3 = \frac{I_R}{2}$, $I_2 = \frac{I_R}{4}$, etc. Next write the current balance equation for the inverting node of the bottom op-amp. It is

$$\frac{V_o'}{R} + \sum_{i=0}^{3} I_i \left(1 - d_i\right) = 0$$

where we name the output of the lower op-amp V_o'. The current balance equation at the inverting node of the top op-amp looks like this

$$\frac{V_o}{R} + \frac{V_o'}{R} + \sum_{i=0}^{3} I_i d_i = 0$$

Eliminating V_o' between these two equations we get

$$\frac{V_o}{R} + \sum_{i=0}^{3} I_i d_i - \sum_{i=0}^{3} I_i \left(1 - d_i\right) = 0$$

$$\begin{aligned}\frac{V_o}{R} &= \sum_{i=1}^{3} I_i \left(1 - d_i\right) - I_i d_i \\ &= \sum_{i=1}^{3} I_i (1 - 2d_i) \\ &= \sum_{i=1}^{3} \frac{I_R}{2^{4-i}} \left(1 - 2d_i\right)\end{aligned}$$

$$\begin{aligned}V_o =& V_R \sum_{i=0}^{3} \frac{1 - 2d_i}{2^{4-i}} \\ =& \frac{V_R}{2^4} \sum_{i=0}^{3} \left(1 - 2d_i\right) 2^i \\ =& \frac{V_R}{16} \sum_{i=0}^{3} \left(1 - 2d_i\right) 2^i\end{aligned}$$

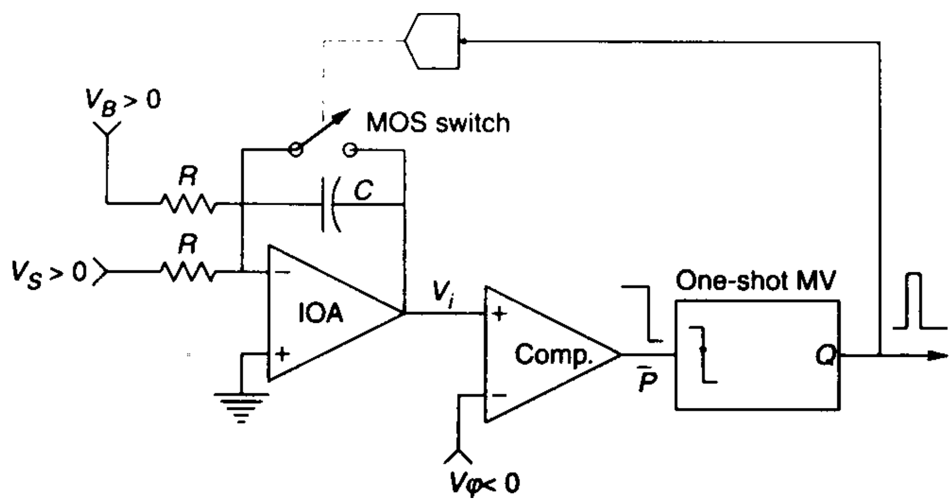
The largest output voltage is when $d_i = 0$. Then,

$$\begin{aligned}V_{o,\max} =& \frac{V_R}{16} \sum_{i=0}^{3} 2^i \\ =& \frac{15}{16} V_R\end{aligned}$$

The smallest output voltage is when $d_i = 1$. Then,

$$V_{o,\min} = \frac{V_R}{16}\sum_{i=0}^{3}(-1)\times 2^i$$
$$= -\frac{15}{16}V_R$$

9.5



(a) We analyze the integrator with

$$\frac{V_B}{R} + \frac{V_s}{R} + \frac{sCV_i}{=}0$$

or

$$sV_i = -\frac{V_B + V_s}{RC}$$

In time-domain notation it is

$$\frac{dV_i}{dt} = -\frac{V_B + V_s}{RC}$$

Assuming V_B and V_s are constant, we can write

$$V_i(t + t_0) = V_i(t_0) - (t - t_0)\,\frac{V_B + V_s}{RC}$$

Now whenever $V_i(t) = V_\phi$, the MOS switch closes and $V_i(t + \epsilon) = 0$. The time interval for this to happen is, Δt, can be found from

$$V_\phi = -\Delta t\frac{V_B + V_s}{RC}$$

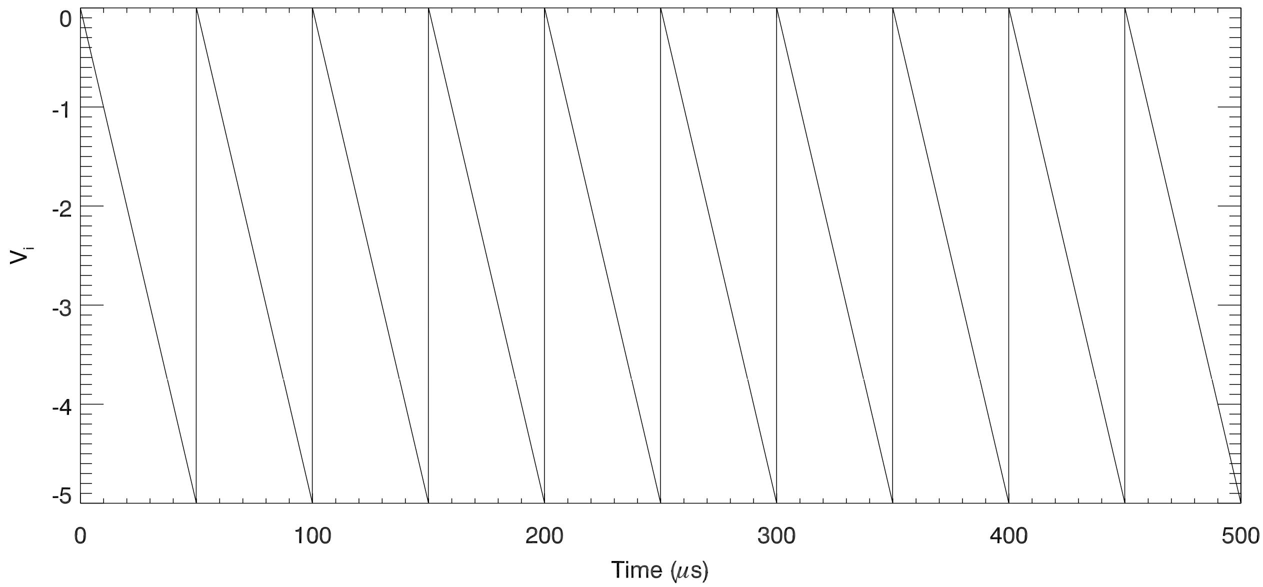
or

$$\Delta t = -RC\frac{V_\phi}{V_B + V_s}$$

V_i is thus piecewise linear in intervals of length Δt, starting at time $t_0 = n\Delta t$, with the expression

$$V_i(t) = -\left(t - t_0\right)\frac{V_B + V_s}{RC}$$

and $t \in [t_0; t_0 + \Delta t[$. For $R = 10\,\mathrm{k}\Omega$, $C = 1\,\mathrm{nF}$, $V_\phi = -5\,\mathrm{V}$, $V_B = 0$, and $V_s = 1\,\mathrm{V}$, the function is plotted below.



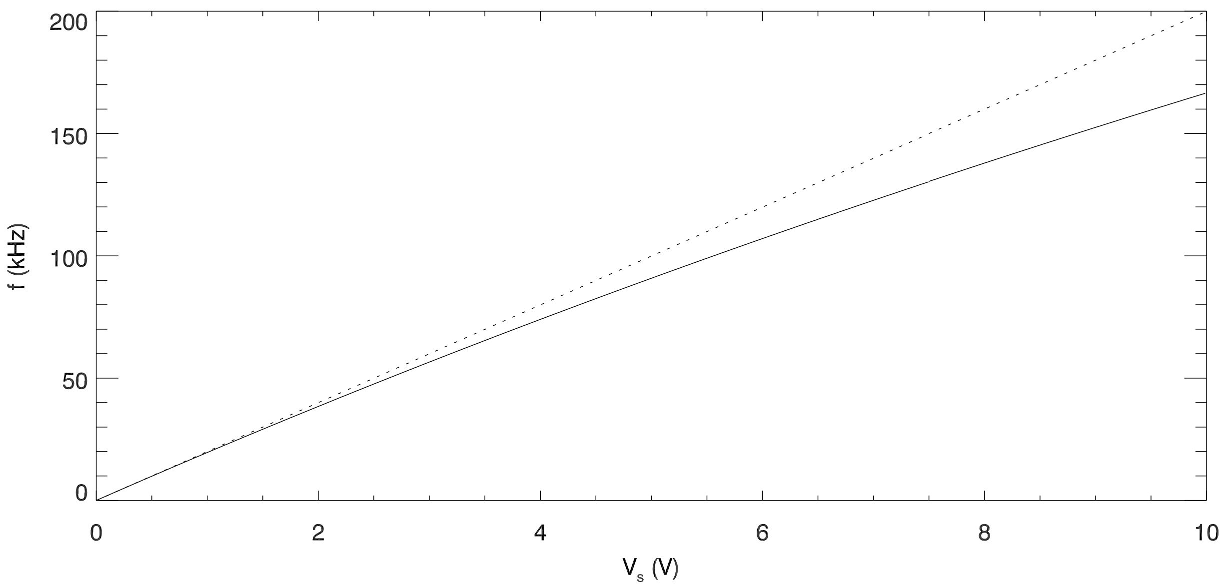
(b) We want to plot the frequency of output pulses as a function of $0 \leq V_s \leq 10\,\mathrm{V}$ when $V_B = +5\,\mathrm{V}$. The output pulses have width $\delta t = 1\,\mu\mathrm{s}$. The time between pulses is

$$T_{\mathrm{pulse}} = \Delta t + \delta t$$

and the frequency of pulses is

$$f_{\mathrm{pulse}} = \frac{1}{\Delta t + \delta t}$$

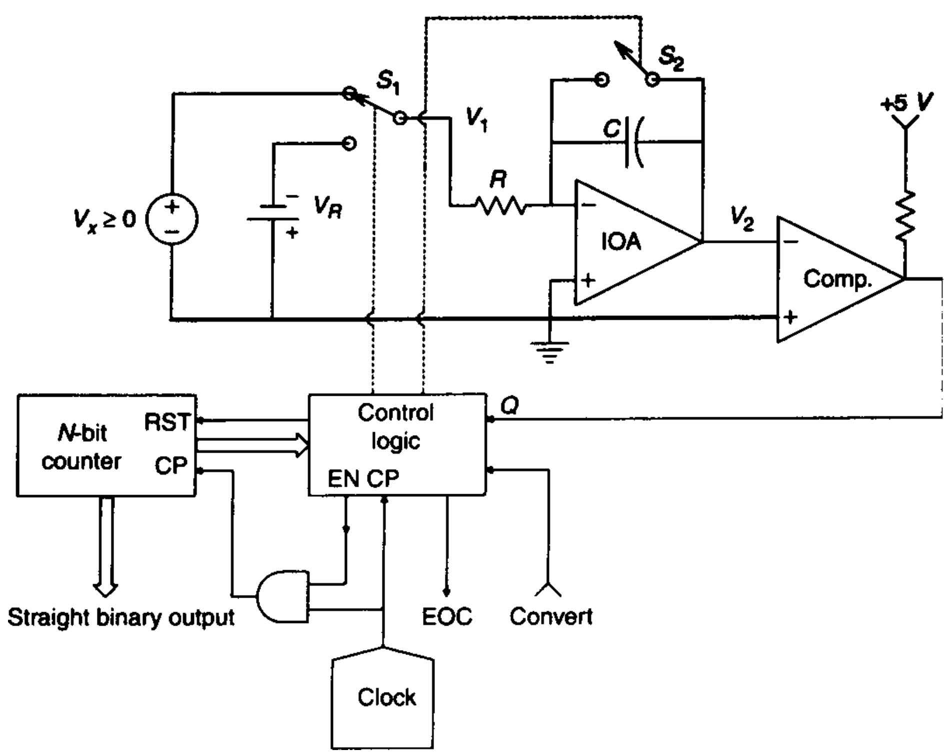
Inserting the previously derived expression for Δt we get the following plot. Note that the curve (solid) is curved. The dotted curve shows the same for the case of zero-width pulse.



(c) We are to find the length of time, T_c, that it will take to count $n = 10^4$ pulses when $V_s = 10\,\text{V}$. This time is simply

$$T_c = \frac{n}{f_{\text{pulse}}} = n\,(\Delta t + \delta t) = 0.06\,\text{s}$$

9.8



We are asked to choose V_R, R, C, and T_C when $N = 10$ (10-bit system), such that the total conversion time is less than 1 second. We are also told that the output, $M = 1000$ when $V_X = 10\,\text{V}$.

Let's start with the conversion time first

$$\left(2^N + M\right) T_C \leq 1\,\mathrm{s}$$

Since the largest value of $M = 2^N - 1$, we find that

$$\left(2^N + 2^N - 1\right) T_C \leq 1\,\mathrm{s}$$

So let's choose

$$T_C = \frac{1\,\mathrm{s}}{2^{N+1}} = \frac{1}{2^{11}}\,\mathrm{s} \approx 0.00049\,\mathrm{s}$$

Next let us find V_R. We are told that when $V_X = 10\,\mathrm{V}$, we want $M = 1000$. The peak integrated voltage, $V_{2,\mathrm{peak}}$, is

$$V_{2,\mathrm{peak}} = KMV_R = K2^N V_x$$

Where K is a yet-to-be-determined constant. From that we can extract

$$V_R = \frac{2^N}{M} V_x$$

For $V_x = 10\,\mathrm{V}$ and $M = 1000$,

$$V_R = \frac{1024}{1000} V_x = 10.24\,\mathrm{V}$$

Finally we want to determine R and C. Let's start by first determining the time constant RC. I am going to design it such that the peak output voltage of the integrating op-amp, V_2 is at most equal to $-V_R$. We find that the peak value of $V_{2,\mathrm{peak}}$ is

$$V_{2,\mathrm{peak}} = -\frac{2^N T_c}{RC} V_x$$

and the largest value of $V_x = V_R$. So

$$-V_R = -\frac{2^N T_c}{RC} V_R$$

$$RC = 2^N T_c = \frac{2^{10}}{2^{11}}\,\mathrm{s} = \frac{1}{2}\,\mathrm{s}$$

R is also the input resistance, so let's make it large, $R = 10\,\mathrm{M\Omega}$. Then

$$C = \frac{1}{2}\frac{1}{10^7}\,\mathrm{F} = 50\,\mathrm{nF}$$