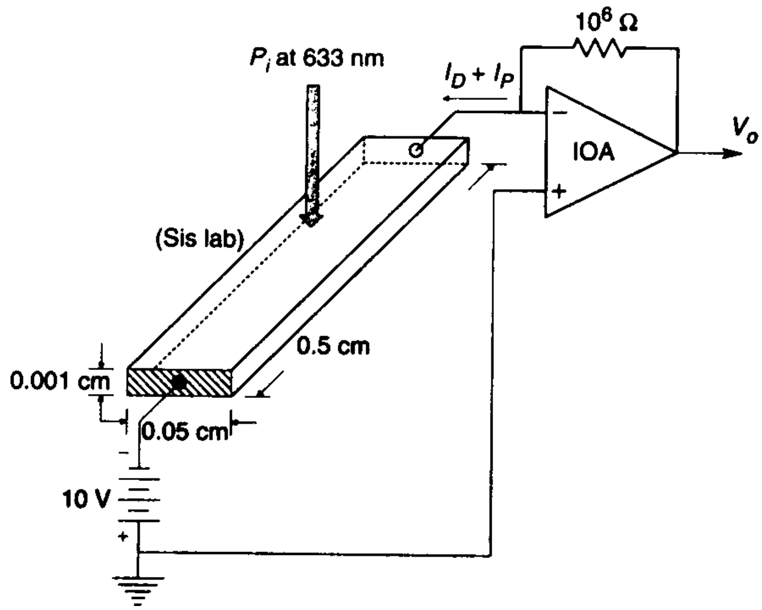
EE 521 Instrumentation and Measurements
Fall 2007
Solutions for homework assignment #7

6.2



(a)
The resistance of a conductor with length L, cross-sectional aread A, and resistivity ρ is

$$R = \frac{\rho L}{A}$$

In this case we have $L = 0.5\,\mathrm{cm}$, $A = 0.001 \times 0.05\,\mathrm{cm}^2$, and $\rho = 2.30 \times 10^3\,\Omega\,\mathrm{cm}$, so we get

$$R = \frac{2.30 \times 10^3 \times 0.5}{0.001 \times 0.05} = 23\,\mathrm{M}\Omega$$

The dark current is then

$$I_D = \frac{V_s}{R} = \frac{10\,\mathrm{V}}{23\,\mathrm{M}\Omega} = 0.435\,\mu\mathrm{A}$$

The output voltage is

$$V_o = I_D R_F = 0.435\,\mu\mathrm{A} \times 10^6\,\Omega = 0.435\,\mathrm{V}$$

(b)
The photoconductance of the slab of silicon can be modeled as

$$G_P = \frac{q\eta\tau_p\left(\mu_p + \mu_n\right)}{L^2}\left(\frac{P_i\lambda}{hc}\right)$$

We want to know P_i such that the photocurrent equals the dark current,

$$I_p = V_s G_P = I_D = \frac{V_s}{R_D}$$

or

$$G_P = \frac{1}{R_D}$$

Isolating P_i we get

$$P_i = \frac{1}{R_D} \frac{L^2}{q\eta\tau_p\left(\mu_p + \mu_n\right)} \frac{hc}{\lambda}$$

Inserting $R_D = 23\,\mathrm{M\Omega}$, $L = 0.5\times10^{-2}\,\mathrm{m}$, $q = 1.60\times10^{-19}\,\mathrm{C}$, $\eta = 0.8$, $\mu_p = 450\times10^{-4}\,\mathrm{m^2/(V\cdot s)}$, $\mu_n = 1500\times10^{-4}\,\mathrm{m^2/(V\cdot s)}$, $\tau_p = 10^{-4}$, $h = 6.624\times10^{-34}\,\mathrm{J\cdot s}$, $c = 3\times10^{8}\,\mathrm{m/s}$, and $\lambda = 6.33\times10^{-7}\,\mathrm{m}$ we get

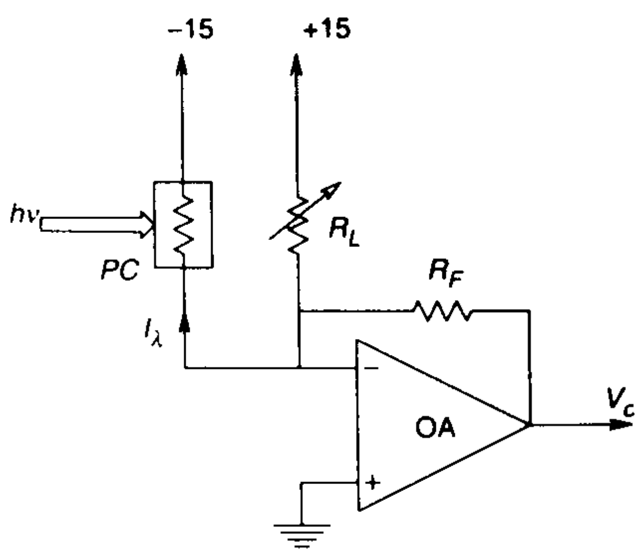
$$\begin{aligned} P_i =& \frac{1}{23\times10^6} \frac{\left(0.5\times10^{-2}\right)^2}{1.6\times10^{-19}\times0.8\times10^{-4}\times\left(450\times10^{-4}+1500\times10^{-4}\right)} \frac{6.624\times10^{-34}\times3\times10^8}{6.33\times10^{-7}} \\ =& 1.37\times10^{-7}\,\mathrm{W} = 0.137\,\mu\mathrm{W} \end{aligned}$$

Per unit aread the incident power is

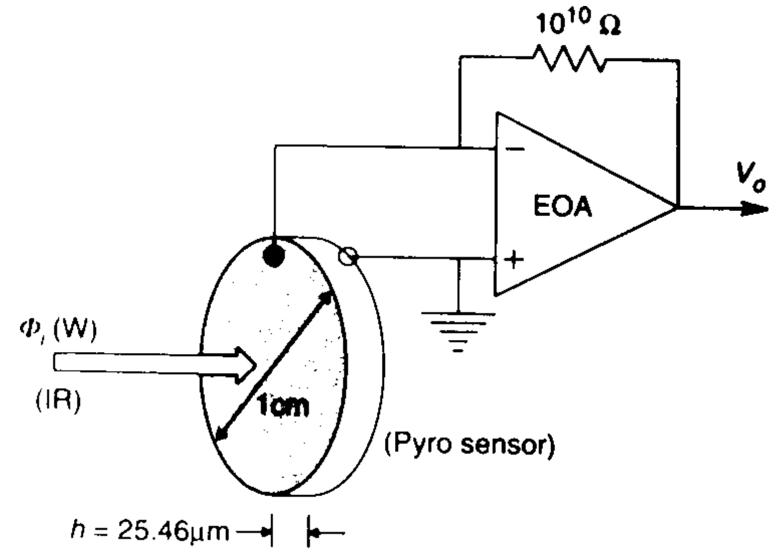
$$\frac{P_i}{A_i} = \frac{P_i}{Lw} = \frac{0.137\times10^{-6}}{0.05\times0.5} = 5.48\times10^{-6}\,\mu\mathrm{W/cm^2}$$

(c)

Bias the inverting input with a current which is the negative of the dark current, as explained in in the figure below, Figure 6.6 from Northrop.



6.3



(a)
The sensor is a first-order system. A first order system has the transfer function

$$\frac{V_o(s)}{V_i(s)} = \frac{K}{s + \frac{1}{\tau}}$$

where τ is the time-constant. If we compare this expression to Northrop equation 6.46,

$$\frac{\Delta T(s)}{P_i(s)} = \frac{\Theta}{s\Theta C_T + 1} = \frac{\frac{1}{C_T}}{s + \frac{1}{\Theta C_T}}$$

We see that

$$\begin{aligned}\tau =& \Theta C_T = \Theta c A h = \Theta c \pi \left(\frac{D}{2}\right)^2 h \\ =& 200\,\mathrm{K/W} \times 2.34 \times 10^6\,\mathrm{J}/\left(\mathrm{m}^3 K\right) \times \pi\left(0.5 \times 10^{-2}\right)^2 \times 25.46 \times 10^{-6} \\ =& 0.936\,\mathrm{s}\end{aligned}$$

(b)
The transfer function from incident power to current is

$$\frac{I_P}{P_i} = \frac{sK_pA\Theta}{s\Theta C_T + 1}$$

And the transfer function from current to output voltage is

$$\frac{V_o}{I_P} = R_F$$

The Laplace transform of the output voltage is therefore

$$V_o(s) = R_F I_P = R_F \frac{sK_pA\Theta}{s\Theta C_T + 1} P_i$$

The incident power is a square pulse whose Laplace transform is

$$P_i(s) = \Phi_o \frac{1 - e^{-sT}}{s}$$

Inserting we get

$$\begin{aligned}
V_o(s) =& R_F \frac{s K_p A \Theta}{s \Theta C_T + 1} \Phi_0 \frac{1 - e^{-sT}}{s} \\
=& R_F \Phi_0 \frac{\left(1 - e^{-sT}\right) K_p A \Theta}{s \Theta C_T + 1} \\
=& R_F \Phi_0 K_p A \Theta \left[\frac{1}{s \Theta C_T + 1} - \frac{e^{-sT}}{s \Theta C_T + 1} \right] \\
=& \frac{R_F \Phi_0 K_p A}{C_T} \left[\frac{1}{s + \frac{1}{\Theta C_T}} - \frac{e^{-sT}}{s + \frac{1}{\Theta C_T}} \right]
\end{aligned}$$

To get the time-series, $V_o(t)$, we inverse Laplace transform

$$V_o(t) = \frac{R_F \Phi_0 K_p A}{C_T} \left[\exp\left(-\frac{t}{\Theta C_T} \right) u(t) - \exp\left(-\frac{t - T}{\Theta C_T} \right) u(t - T) \right]$$

Using $\tau = \Theta C_T$ it simplifies to

$$V_o(t) \frac{R_F \Phi_0 K_p A}{C_T} \left[e^{-\frac{t}{\tau}} u(t) - e^{-\frac{t-T}{\tau}} u(t - T) \right]$$

Where $\tau = 0.936\,\mathrm{s}$, $R_F = 10^{10}\,\Omega$, $\Phi_0 = 100\,\mathrm{nW}$, $K_p = 60\,\mu\mathrm{C}/\left(\mathrm{m}^2 K\right)$, $A = \pi \left(\frac{D}{2}\right)^2 = \pi 0.005^2\,\mathrm{m}^2$, $C_T = cAh$, $c = 2.34 \times 10^6\,\mathrm{J}/(m^3 K)$, $h = 25.46\,\mu\mathrm{m}$,and $T = 3\,\mathrm{s}$. The function is ploted below