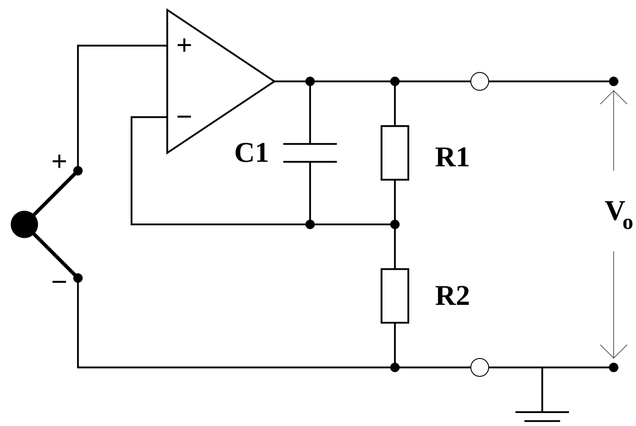
EE 521 Instrumentation and Measurements
Fall 2007
Computational Laboratory #1

Introduction

In this exercise you will analyze a thermocouple used to measure microthermal atmospheric turbulence. Atmospheric turbulence is manifested as small-scale and rapid variations of temperature and density, and thus index of refraction. Variations in the index of refraction causes diffraction of light passing through the atmosphere, and that degrades the performance of optical instruments performing measurements through the atmosphere. By measuring rapid variations of temperature in the atmosphere we are able to characterize atmospheric turbulence. The following figure shows a simple circuit which conditions the output of a thermocouple for input into an analog to digital converter.



The thermocouple is a CO-2K manufactured by Omega Engineering, the amplifier is a OP-27, $C_1 = 0.047\,\mu$F, $R_1 = 220\,$KΩ, and $R_2 = 22\,\Omega$. The thermocouple is soldered to a small PC board which holds the OP-27. The signal V_o is sampled by an analog-to-digital converter (ADC) with a full range of $[-5; +5]$ V. The ADC records 300 samples per second. The supplied data file contains a sequence of samples of V_o recorded while the thermocouple is measuring the atmosphere. The values in the data file are 4-byte floating point values and are in volts.

Assignment

1. Explain what this sensor really measures. “It measures temperature” is not sufficient. It measures a temperature difference, but the difference between what two temperatures?

2. We are interested in the RMS variation of air temperature over a minute at a time, not how it varies over hours, during the day, or seasonally. Explain how this circuit with a single thermocouple allows us to use a very high gain and a full-scale range much smaller than daily and seasonal temperature variations, and still obtain readings in range. What is the full-scale range of temperature?

3. Argue that a first-order model for the relationship between air-temperature, T, and electrical output, V_T, of the thermocouple is reasonable. Then write the first order differential equation for the thermocouple, and then the equivalent transfer function.

4. Derive the transfer function for the remainder of the circuit, with the thermocouple voltage, V_T, as input, and V_o as output. Then write the transfer function for V_o/T.

5. What is the Gain of the amplifier circuit? Is it capable of providing this gain over the sample bandwidth? How large a bandwidth can it provide this gain over?

6. What is the purpose of the capacitor?

7. Plot the bode plot (both amplitude and phase) for the transfer function of V_o/T. Assume a time-constant for the thermocouple of 0.05 s.

8. Derive an expression for the noise power spectrum at the output V_o, assuming that the amplifier has a input noise e_{na}, as specified in the data sheet for the OP-27 (if you find more than one value for e_{na}, pick a representative value). Assume the thermocouple produces no noise.

9. What is the RMS noise voltage at the output V_o?

10. What is the SNR at the output if the temperature varies sinusoidally at low frequency with an amplitude of 1 K, 0.1 K, 0.01 K? What is the noise-limited temperature resolution?

11. Plot a short segment of the provided data.

12. Plot the auto-correlation function of the data.

13. Plot the power spectrum of the data. When you compute the power spectrum you may benefit from cutting the data set into shorter segments, computing the power spectrum from each one, and then averaging the power spectra. This will result in a power spectrum with fewer data points, but each data point will have better SNR. Figure 2 of Short et al. goes further and averages data points such that they are evenly spaced on a logarithmic scale. This further improves the visual appearance of the power spectrum. You are welcome to do that as well, but not required to.

14. Derive equation 1 in Short et al. using your knowledge of the transfer function, and assuming that the air temperature approximately follows a Kolmogorov power spctrum. If you end up with a more complex expression, explain why you can simplify it to look like the one in Short et al. (Hint: sampling frequency).

15. Notice the knee in the power spectrum. It tells you the time-constant of the thermocouple. Explain why, and estimate the time-constant of the thermocouple. You may do a least-squares fit if you like, or simply estimate it by overplotting the model with different parameters until it fits. Why is the time-constant much longer than that specified by Omega Engineering (Hint: thermal mass)?

16. Notice the flattening of the spectrum at high frequency. Assume this is white noise (why do we assume white noise eventhough we found that the noise at V_o is not white?), estimate the root power of the noise (in $V/\sqrt{Hz}$) from the power spectrum. Is this noise consistent with the electronic noise you estimated at V_o?

17. Where do the two high-frequency spikes in the power spectrum come from?
18. Assuming that the noise is due to the digitization, with the full range of the ADC as specified earlier, what is a guess for the number of sigifnicant bits of output from the ADC? Refer to Northrop Section 9.3.
19. Verify your answer in the previous question by looking carefully at the data in the data file. You will notice that it is discretized.

    Extra credit questions:

20. The time-constant of the thermocouple corresponds to a sample frequency significantly smaller than the sample frequency used. Does it make sense to use a high sample frequency (pros and cons)? What can be done to correct the data to attempt to recover the high-frequency portion of the power spectrum of air temperature given the measured power spectrum of voltage?
21. Is the slew rate of the OP-27 sufficiently high for this use?

References

1. Northrop Section 6.4.1 describes thermocouples.
2. Northrop Section 9.3 describes the equivalent noise of digitizing data.
3. Short et al., The Astrophysical Journal, Volume 599, page 1469-1477, 2003. Discusses a similar experiment carried out at Mount Wilson observatory. The model for the power spectrum given at the beginning of the paper is useful, and should be derived if you use it.
4. Specifications sheet for Omega Enineering CO-2K Style 2 thermocouple. Describes the geometry and electrical characteristics of the thermocouple. You can search for it at www.omega.com
5. Data sheet for the OP-27 op-amp. You can search for it at www.digikey.com