Solutions to homework #1 due 2007/1/30

Problem 1
The index of refraction of a medium is defined as the ratio of the speed of light in vacuum to the speed of light in the medium,

$$n = \frac{c}{v}$$

We have $c = 3 \times 10^{10}\ \frac{\text{cm}}{\text{s}}$, and $v = 2 \times 10^{10}\ \frac{\text{cm}}{\text{s}}$, so that

$$n = \frac{c}{v} = \frac{3 \times 10^{10}\ \frac{\text{cm}}{\text{s}}}{2 \times 10^{10}\ \frac{\text{cm}}{\text{s}}} = 1.5$$

Problem 2
Again, we have the formula

$$n = \frac{c}{v}$$

which we can re-arrange to

$$v = \frac{c}{n}$$

Inserting $n = 1.33$ and $c = 3 \times 10^{10}\ \frac{\text{cm}}{\text{s}}$ we get

$$v = \frac{3 \times 10^{10}\ \frac{\text{cm}}{\text{s}}}{1.33} = 2.3 \times 10^{10}\ \frac{\text{cm}}{\text{s}}$$

Problem 3
For this problem we need to use Snell's law,

$$n_1 \sin I_1 = n_2 \sin I_2$$

which we can re-arrange as

$$I_2 = \sin^{-1}\left(\frac{n_1}{n_2} \sin I_1\right)$$

The angle of insidence in medium 1 is 30°.

(a) Medium 1 is air, $n_1 = 1$, and medium 2 is glass with $n = 1.5$

$$I_2 = \sin^{-1}\left(\frac{1}{1.5} \sin 30^\circ\right) = 19.5^\circ$$

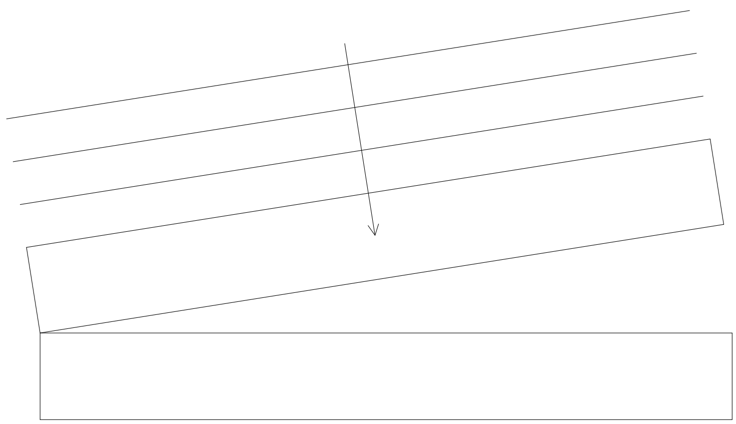
(b) Medium 1 is water, $n_1 = 1.33$, and medium 2 is air, $n = 1$

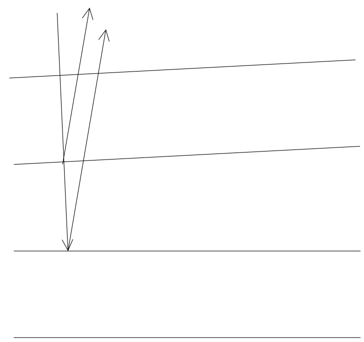
$$I_2 = \sin^{-1}\left(\frac{1.33}{1} \sin 30^\circ\right) = 41.7^\circ$$

(c) Medium 1 is water, $n_1 = 1.33$, and medium 2 is glass, $n = 1.5$

$$I_2 = \sin^{-1}\left(\frac{1.33}{1.5}\sin 30^\circ\right) = 26.3^\circ$$

Problem 4
The setup is as follows



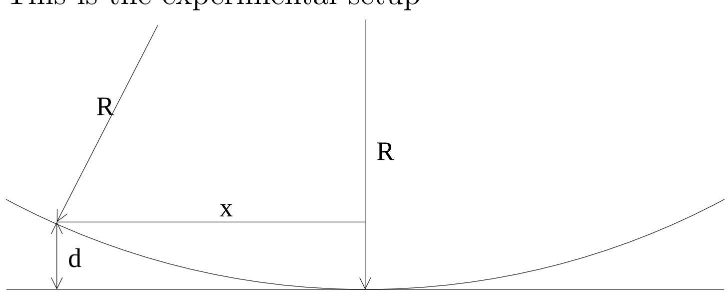


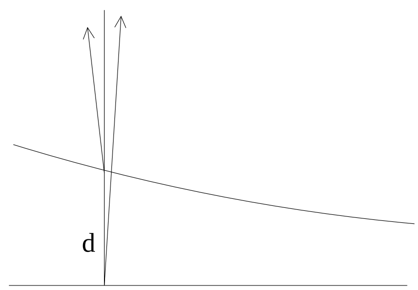
There will be a bright fringe when the distance between the top and bottom flats times two (light passes through the space twice) is equal to an integer number of wavelengths. Over 6 inches the distance between the plates changes by 0.003 inches. The wavelength of the light is 0.00002 inches. Twice the maximum distance, 0.006 inches, corresponds to 0.006/0.00002=300 wavelengths. So we see 300 fringes across the flat.

Problem 5

If we put water between the two flats instead of air, the wavelength of the light changes. As the speed of the light in water is reduced by a factor of n relative to air, the wavelength is also reduced by a factor of n. So the maximum extra distance traveled corresponds to 0.006/(0.00002/1.3333)=400 wavelenghts. So we see 400 fringes across the flat.

Problem 6
This is the experimental setup





The relationship between d and x is

$$(R - d)^2 + x^2 = R^2$$

or

$$
\begin{aligned}
x^2 &= R^2 - (R-d)^2 \\
x &= \sqrt{R^2 - (R-d)^2} \\
&= \sqrt{R^2 - R^2 - d^2 + 2dR} \\
&\approx \sqrt{2dR}
\end{aligned}
$$

There is a 180° phase shift a the lower reflection point. We thus have dark bands when the extra path traveled, $2d$ is equal to a whole number of wavelengths, because then the two waves will be out of phase,

$$2d = n\lambda, \quad n \geq 1$$

We need to find x for each dark ring, so we eliminate d

$$x = \sqrt{2dR} = \sqrt{n\lambda R}$$

We assume that $\lambda = 0.000020$ inches, and $R = 20$ inches. We can then compute x. Note that the solutions given in the book are diameters, whereas x is a radius. So we give both

n	x inches	diameter inches
1	0.020	0.040
2	0.028	0.057
3	0.035	0.069

If we increase $R = 200$ inches, then we have the values

n	x inches	diameter inches
1	0.063	0.126
2	0.089	0.179
3	0.110	0.219