Solutions to homework #3 due 2007/2/6

Problem 1
We start with

$$x' = -\frac{f^2}{x} = -\frac{f^2}{s+f}$$

where $f = 10''$ and $s = -200'$.
(a)

$$x' = -\frac{10^2}{-200 \times 12 + 10} = 0.0418$$

(b)
The distance from the second principal point is s', where

$$s' = f + x' = 10 + 0.0418 = 10.0418$$

Problem 2
We have the relation

$$\frac{h'}{h} = \frac{f}{x} = \frac{f}{s+f}$$

(a)

$$h' = \frac{f}{s+f}h = \frac{10}{-200 \times 12 + 10} \times 50 \times 12 = -2.510\text{"}$$

(b)

$$m = \frac{h'}{h} = \frac{-2.510}{50 \times 12} = -0.00418$$

Problem 3
We use

$$\frac{1}{s'} = \frac{1}{s} + \frac{1}{f}$$

with $s = -20$, and $f = -5$. We get

$$s' = \frac{1}{\frac{1}{s} + \frac{1}{f}} = \frac{1}{\frac{1}{-20} + \frac{1}{-5}} = -4$$

So the image is 4 inches to the left of the second principal plane. This is a virtual image. The height and width of the image are both computed from the formula

$$h = w = 1\text{"} \times m = 1\text{"} \times \left(-\frac{f}{x}\right) = -1\text{"} \times \frac{f}{s+f} = -\frac{-5}{-20-5} = 0.2\text{"}$$

The thickness is

$$t = 1" \times \bar{m} = 1" \times m^2 = \left(\frac{f}{s+f}\right)^2 = 0.2^2 = 0.04"$$

Problem 4
Use

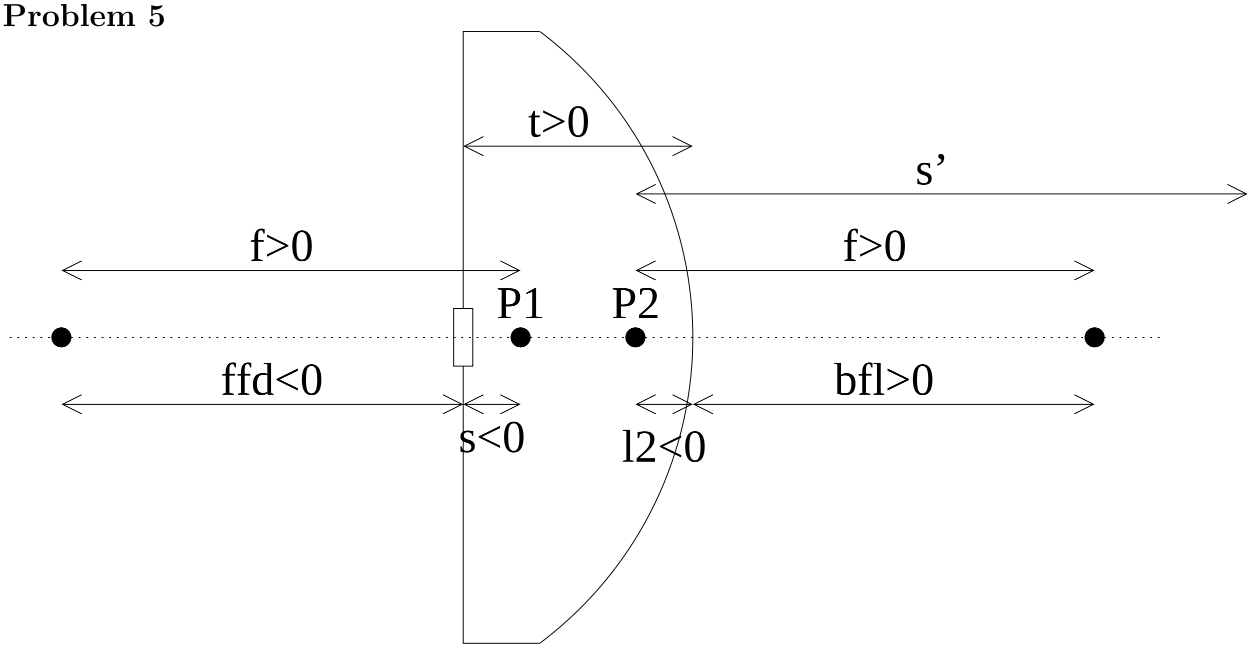
$$\frac{1}{s'} = \frac{1}{f} + \frac{1}{s}$$

with $f = 2$, and $s = -1$. We get $s' = -2$, so that the image is located 2 inches to the left of the second principal point. This is virtual image. For the magnification we use

$$m = \frac{f}{x} = \frac{f}{s+f}$$

with $f = 2$ and $s = -1$. We get $m = 2$. Positive magnification which is consistent with it being a virtual image.

Problem 5



We first need to compute the locations of the principal points. We will need the location of the first principal point to know what s is, and we will need to know the second principal point because the image is located relative to it.

From the figure we can see that

$$\text{ffd} + s = -f \;\Rightarrow\; s = -f - \text{ffd} \;\text{ and }\; l_2 = \text{bfl} - f$$

We need the expressions for f, bfl, and ffd,

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + t\frac{n-1}{R_1 R_2 t}\right] \qquad \text{bfl} = f - \frac{ft\,(n-1)}{nR_1} \qquad \text{ffd} = -f + \frac{ft\,(n-1)}{nR_2}$$

Notice, that since $R_1 = \infty$, bfl $= f$, and P_2 is located at the second surface. This means that the s' that we get is relative to the second surface. Also, notice that $R_1 = \infty$. Finally, we can insert the expression for ffd in the expression for s. We can re-write

$$\frac{1}{f} = -\frac{n-1}{R_2} \qquad s = -\frac{ft\,(n-1)}{nR_2}$$

f is independent of t and plugging in numbers we get $f = 20\,\text{mm}$. We relate the location of the object and the location of the image by

$$\frac{1}{s'} = \frac{1}{f} + \frac{1}{s}$$

Finally, the size of the image is computed as

$$h' = hm = h\frac{f}{x} = \frac{fh}{f+s}$$

Now we are ready to plug in numbers
(a)
$t = 7$, so we get $s = -4.\bar{6}$, $s' = -6.09$. The image is thus located 6.09 mm behind the curved surface. The height of the image is $h' = 1.30\,\text{mm}$.
(b)
$t = 10$, so we get $s = -6.\bar{6}$, $s' = -10$. The image is located 10 mm behind the curved surface. The height of the image is $h' = 1.5\,\text{mm}$.
(c)
$t = 16.67$, so we get $s = -11.11\bar{3}$, $s' = -25.01$. The image is located 11.11 mm behind the curved surface. The height of the image is $h' = 2.25\,\text{mm}$.

Problem 6

We will use the paraxial ray tracing equations

$$n'u' = nu - y\frac{n'-n}{R} \qquad y_{n+1} = y_n + t_n\frac{n'_n u'_n}{n_{n+1}}$$

There are two surfaces, $R_1 = 100$, $R_2 = -100$. The thickness is $t = 10$. I will use the tabular ray tracing form of Figure 2.1, page 38.

		Surface #1		Surface #2	
Radius		100		-100	
Thickness			10		
Index	1.0		1.5		1.0

Ray trace 1

y		1.0		$0.9\bar{6}$	
nu	0.0		-0.005		$-0.0098\bar{3}$

Ray trace 2

y		10		$9.\bar{6}$	
nu	0.0		-0.05		$-0.098\bar{3}$

Problem 7
For the effective focal length, f, we have

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + t\frac{n-1}{R_1 R_2 n}\right]$$

Inserting $R_1 = 100$, $R_2 = -100$, $t = 10$, and $n = 1.5$, we get

$$f = 101.695$$

For the back focal length we have

$$\text{bfl} = f - \frac{ft\,(n-1)}{nR_1}$$

Inserting we get

$$\text{bfl} = 98.3051$$

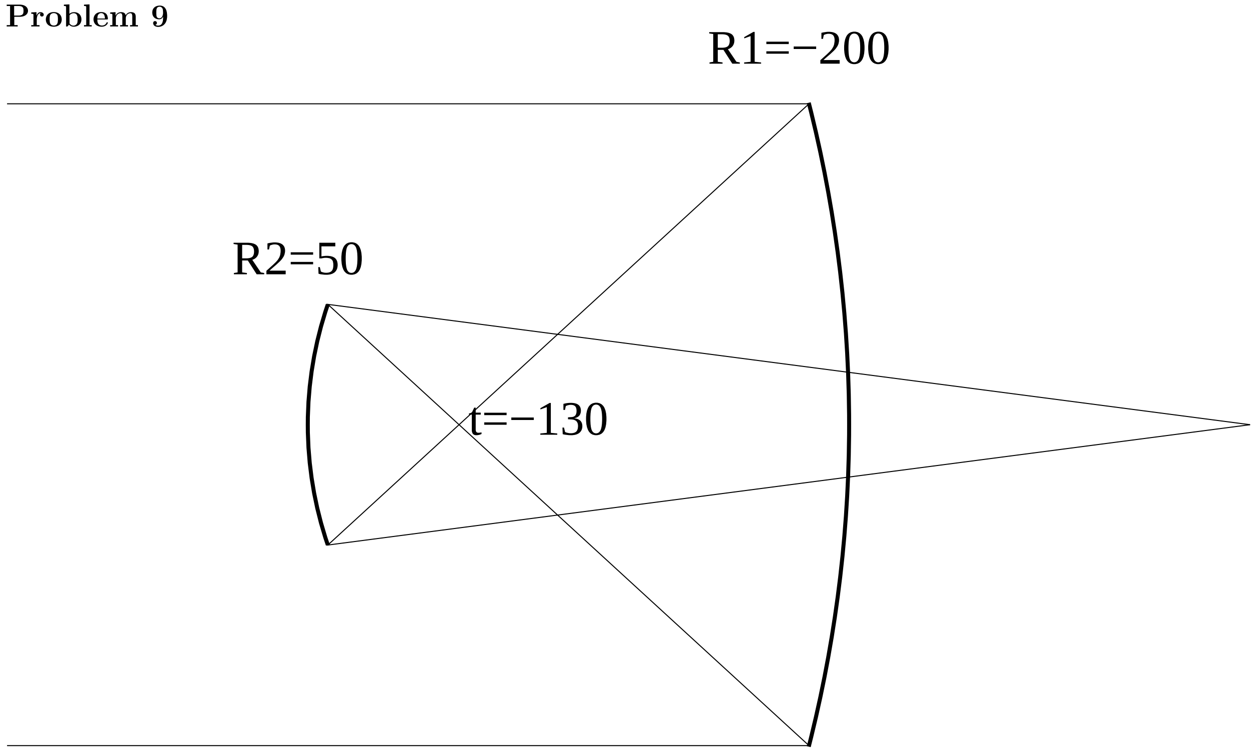
Problem 8
We use the thin lens equation,

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

With $R_1 = 100$, $R_2 = -100$, and $n = 1.5$, and get

$$f = 100.000$$

Problem 9



I will solve this problem by ending a ray through the system parallel to the optical axis, and use it's location and direction as it emerges to compute the focal length and focus location.

		Surface #1		Surface #2	
Radius		-200		50	
Thickness			-130		
Index	1.0		-1.0		1.0
y		1.0		-0.3	
nu	0.0		-0.01		0.002

From this table we can calculate the effective focal length,

$$f = -\frac{y_1}{u_2'} = -\frac{1.0}{0.002} = -500$$

and the back focal length as

$$\text{bfl} = -\frac{y_2}{u_2'} = -\frac{-0.3}{0.002} = 150$$

The back focal length is the distance from the last surface to the focus. Thus the focus is 150 to the right of the secondary, or 20 to the right of the primary.

Problem 10

This is the equation for the effective focal length

$$f_{ab} = \frac{f_a f_b}{f_a + f_b - d}$$

This is the equation for the back focus distance

$$B = \frac{f_{ab}\left(f_a - d\right)}{f_a}$$

And this is the equation for the front focus distance

$$ffd = -\frac{f_{ab}\left(f_b - d\right)}{f_b}$$

Inserting $f_a = 10$", $f_b = -10$", and $d = 5$", we get. $f_{ab} = 20$", $B = 10$", and ffd $= -30$"

Problem 11

We have the requirements that $f_{ab} = 20$, $B = 10$, and $d = 5$. We wish to find f_a and f_b. First isolate f_a in

$$B = \frac{f_{ab}\left(f_a - d\right)}{f_a}$$

$$f_a = \frac{f_{ab}d}{f_{ab} - B} = \frac{20 \times 5}{20 - 10} = 10$$

Next, isolate f_b in

$$f_{ab} = \frac{f_a f_b}{f_a + f_b - d}$$

$$f_b = -\frac{f_{ab}\left(f_a - d\right)}{f_{ab} - f_a} = -\frac{20 \times (10 - 5)}{20 - 10} = -10$$