Solutions to homework #6 due 2007/3/22

Problem 1

To be readable, letters must subtend an angle of at least 5 arcminutes. The 1 mm high letter at distance 300 ft subtends the angle

$$\theta = \tan^{-1}\left(\frac{1\,\text{mm}}{300'}\right) = \tan^{-1}\left(\frac{1\times 10^{-3}}{300\times 0.305}\right) = 1.1\times 10^{-5} = 0.038'$$

The letter must be magnified by a factor of

$$\frac{5'}{0.038'} = 133$$

Problem 2

We normally give a nearsighted person a lens which forms an image at the most distant point that the person can see. The lens is closer to the eye than the most distant point. Therefore, to form an image at that most distant point, the lens would need to be negative. The image of a distant object will be formed at the focal distance of the lens. Therefore, if we choose the lenses focal distance to be $-5\,\text{in}$, the power of that lens is

$$\frac{1}{-5\times 0.0254} = -7.9\,\text{diopters}$$

Problem 3

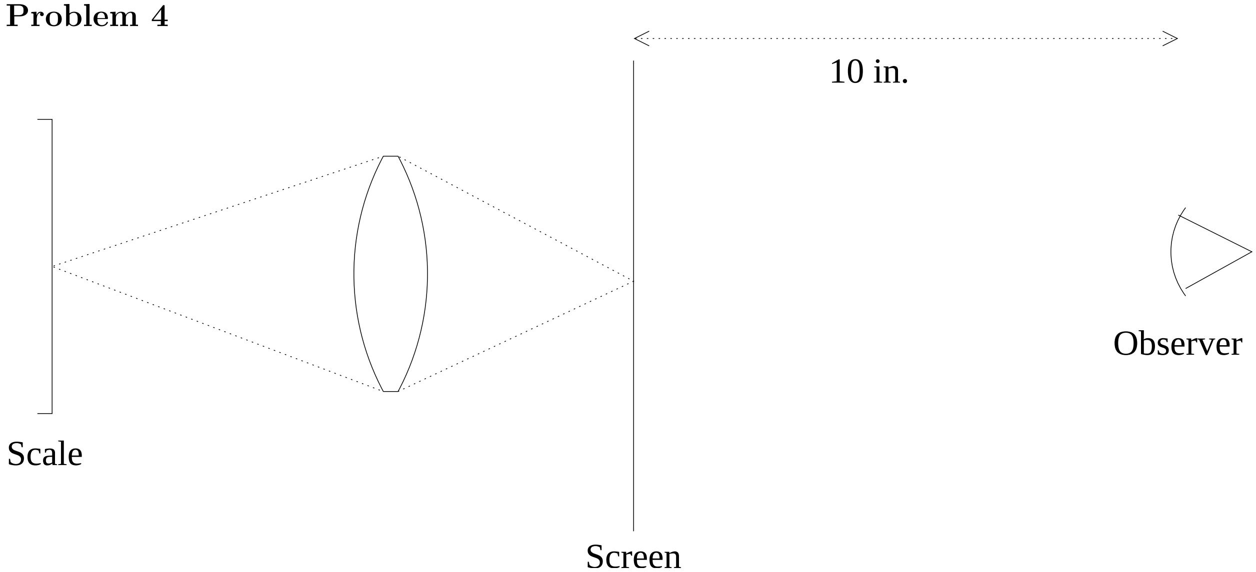
If the eye is focused at $10\,\text{in}$, it is focused at

$$d = \frac{1}{10\times 0.0254} = 3.94\,\text{diopters}$$

If the depth of focus is $\delta = 0.25\,\text{diopters}$, then the range of distances over which objects are in focus is

$$D = \frac{1}{d-\delta} - \frac{1}{d+\delta} = 10.67 - 9.39 = 1.28\,\text{in}$$

Problem 4



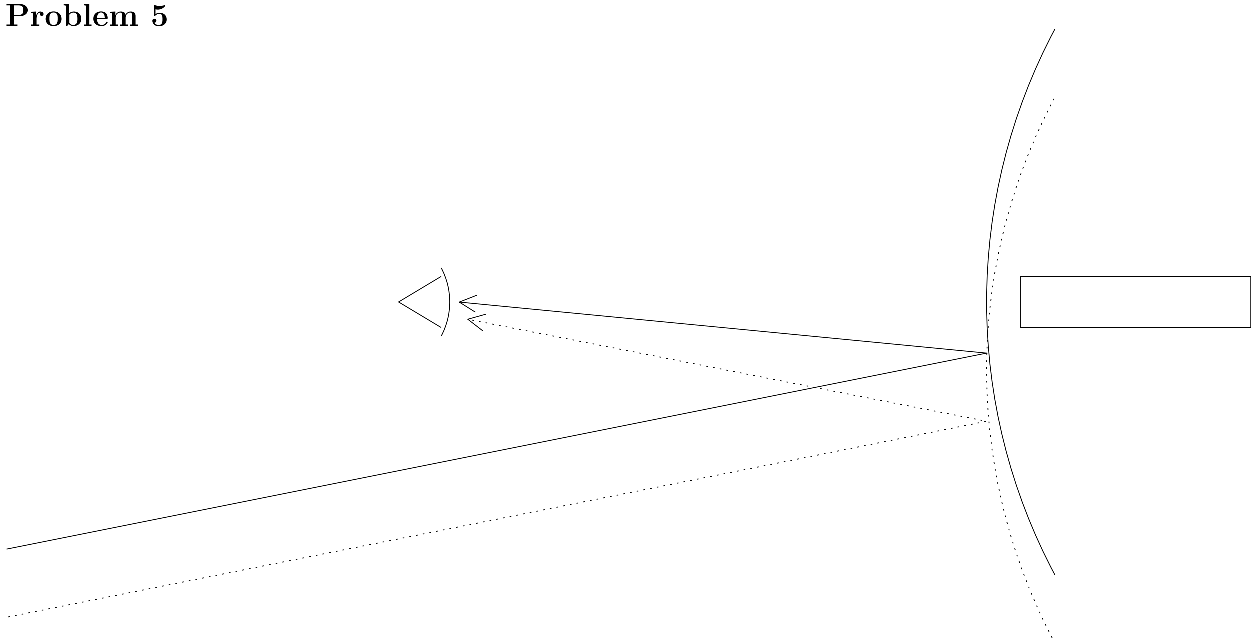
The observer has $\delta\theta = 10$" Vernier acuity. That means that from a distance $d = 10$" the person can distinguish a movement of

$$\delta y = d \tan \delta\theta = 10 \times \tan 4.85 \times 10^{-5} = 0.000485\text{"}$$

on the screen. We must be able to measure a movement of 0.0001" on the scale. Thus the lens must magnify the scale on the screen by a factor of

$$m = \frac{0.000485}{0.0001} = 4.85$$

Problem 5



Because the object is distant, all incoming rays are approximately parallel. If we also assume that the eye of the observer is much larger than the shift we are considering, then the movement of the image observed by the eye is equal to the movement of the mirror.
(a) The smallest amount that the center of curvature can be displaced is therefore equal to the visual acuity of the eye at that distance. If we assume that it is 1 arcminute, then the smallest observable displacement is

$$2\delta y = d \tan \delta\theta = 10\text{"} \times \tan 10\text{"} = 0.00048\text{"}$$

We get the 10" figure from page 131 of Smith from the paragraph beginning "The eye is capable of detecting angular motion..." The offset of the mirror from the center of the spindle is half of the total motion,

$$\delta y = 0.00024\text{"}$$

(b) If the mirror is offset $\delta y = 0.02$", from the center of the spindle, the total angular motion in one rotation is

$$2\delta\theta = \tan^{-1}\left(\frac{\delta y}{d}\right) = 0.11^\circ$$

The slowest motion that the eye can detect is about $\omega_{\text{min}} = 1' - 2', \text{s}^{-1}$., corresponding to spindle rotation rates of

$$f_{\text{min}} = \frac{\omega_{\text{min}}}{2\pi\delta\theta} = \frac{1' - 2'\,\text{s}^{-1}}{2\pi \times 0.11^\circ} = 0.023 - 0.046\,\text{s}^{-1} = 1.4 - 2.8\,\text{min}^{-1}$$

Similarly, the fastest motion that the eye can detect is about $\omega_{\text{max}} = 200^\circ\,\text{s}^{-1}$, corresponding to a spindle rotation rate of

$$f_{\text{max}} = \frac{\omega_{\text{max}}}{2\pi\delta\theta} = \frac{200^\circ\,\text{s}^{-1}}{2\pi \times 0.11^\circ} = 289\,\text{s}^{-1}$$

Problem 6

(a) The relationship between diopters, D, and focal length, f, is

$$f = \frac{1}{D}$$

The range of D is $D \in [-10 \times 10^{-3}; 10 \times 10^{-3}]\,\text{m}^{-1}$, which corresponds to $\pm f \in [100; \infty]\,\text{m}$. The shortest focal length is therefore $f_{\text{min}} = \pm 100\,\text{m}$.

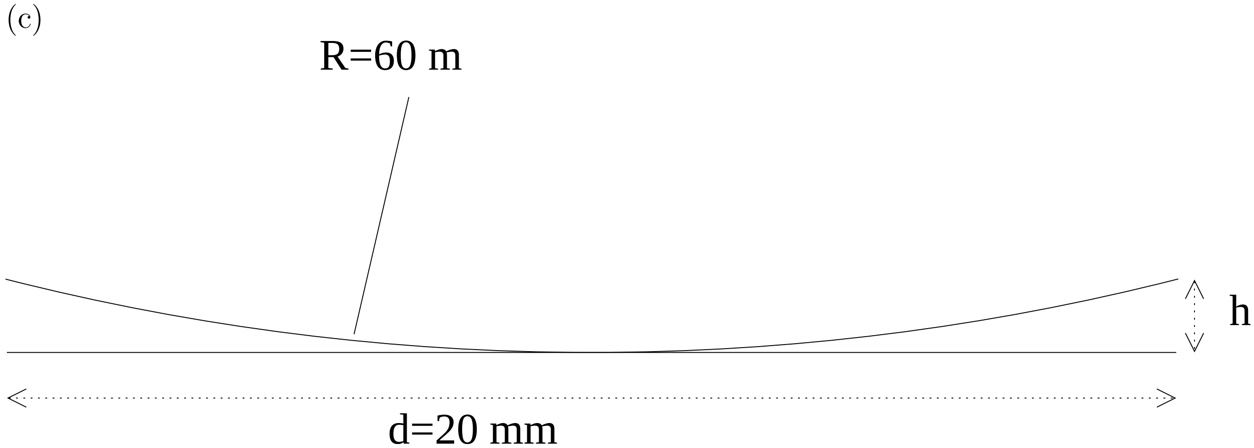
(b) Formula for focal length of a thin lens

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

if $R_1 = \infty$, then

$$R_2 = -(n-1)\,f = \mp(1.6-1)\,100\,\text{m} = \mp 60\,\text{m}$$

(c)



The number of rings, n, is

$$n = \frac{h}{\lambda/2}$$

and h can be computed from

$$h = R - R\cos\theta$$

Let's expand this using $\cos\theta = 1 - \frac{\theta^2}{2}$,

$$h \approx R - R\left(1 - \frac{\theta^2}{2}\right) = \frac{\theta^2}{2}R$$

θ is the angle between the radius which reaches the center and the radius which reaches the edge, so

$$\theta = \tan^{-1}\left(\frac{d/2}{R}\right) \approx \frac{d}{2R}$$

which gives us

$$h = \frac{\theta^2}{2}R = \frac{d^2}{8R} = \frac{\left(20 \times 10^{-3}\right)^2}{8 \times 60} = 8.3 \times 10^{-7}\,\mathrm{m}$$

and the number of rings is

$$n = \frac{h}{\lambda/2} = \frac{2 \times 8.3 \times 10^{-7}}{0.55 \times 10^{-6}} = 3.03$$