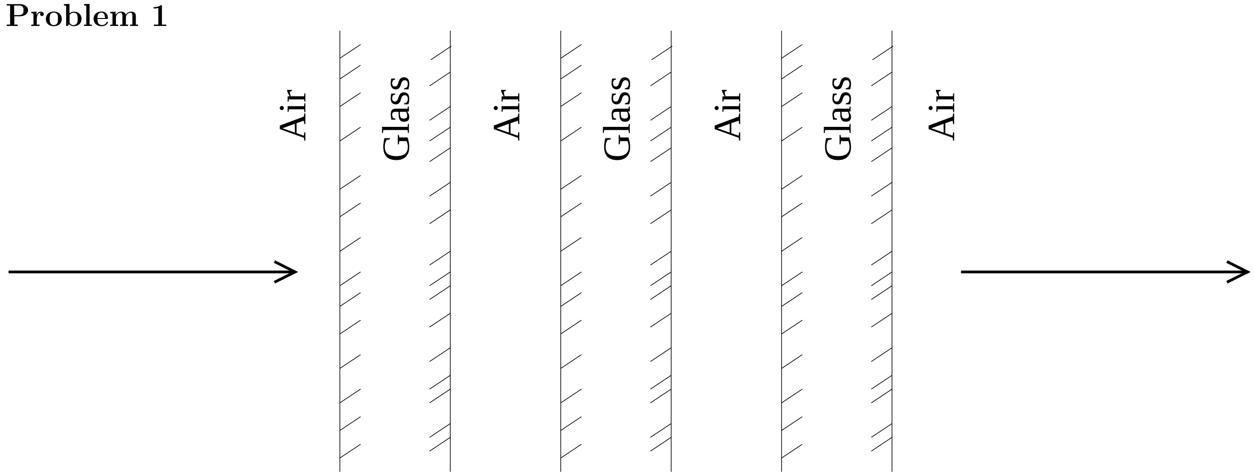
Solutions to homework #8 due 2007/4/10

Problem 1



(a) Since the glass plates are thin we will ignore volume absorption. The effective transmission of a glass plate (effective means including the effects of re-reflection inside the material) is

$$T_t = \frac{T^2}{1 - R^2}$$

where

$$R = \left(\frac{n-1}{n+1}\right)^2 \qquad T = 1 - R$$

The effective transmission through all three glass plates is

$$T_3 = \left(\frac{T^2}{1 - R^2}\right)^3$$

Plugging in $n = 1.5$ we get

$$R = 0.04 \qquad T = 0.96 \qquad T_3 = 0.787$$

(b) At each surface 0.04 of the light is lost, and 0.96 of the light is transmitted directly. Since there are six surfaces, the total direction transmittance is

$$T_6 = T^6 = 0.96^6 = 0.783$$

So the vast majority of the light which makes it through the stack of glass plates is directly transmitted.

Problem 2

(a) When we ignore multiple reflections, then the transmissivity of a thickness of material is

$$T_t = T^2 K$$

where T is the surface transmissivity, and K is the volume transmissivity. For the 1 cm thickness we can write

$$T_{1\,\mathrm{cm}} = T^2 K$$

and for the 1 cm thickness we can write.

$$T_{2\,\mathrm{cm}} = T^2 K^2$$

where we write K^2 for the material which is twice as thick. We can now solve for T and K,

$$K = \frac{T_{2\,\mathrm{cm}}}{T_{1\,\mathrm{cm}}} = \frac{0.8}{0.85} = 0.9412$$

$$T = \frac{T_{1,\mathrm{cm}}}{\sqrt{T_{2,\mathrm{cm}}}} = \frac{0.85}{\sqrt{0.8}} = 0.9503$$

where K is for $1,$cm thickness material. The total transmission for a 3 cm slab is then

$$T_{1,\mathrm{cm}} = T^2 K^3 = 0.9503 \times 0.9412^3 = 0.753$$

(b) For a material thickness x, the absorption coefficient, α, is related to the bulk transmissivity of the material as

$$K = e^{-\alpha x}$$

or

$$\alpha = -\frac{\log K}{x} = \frac{1}{x} \log \frac{1}{K}$$

For $x = 1\,\mathrm{cm}$, $K = 0.9412$, so we get

$$\alpha = \log \frac{1}{0.9412} = 0.061\,\mathrm{cm}^{-1}$$

Problem 3

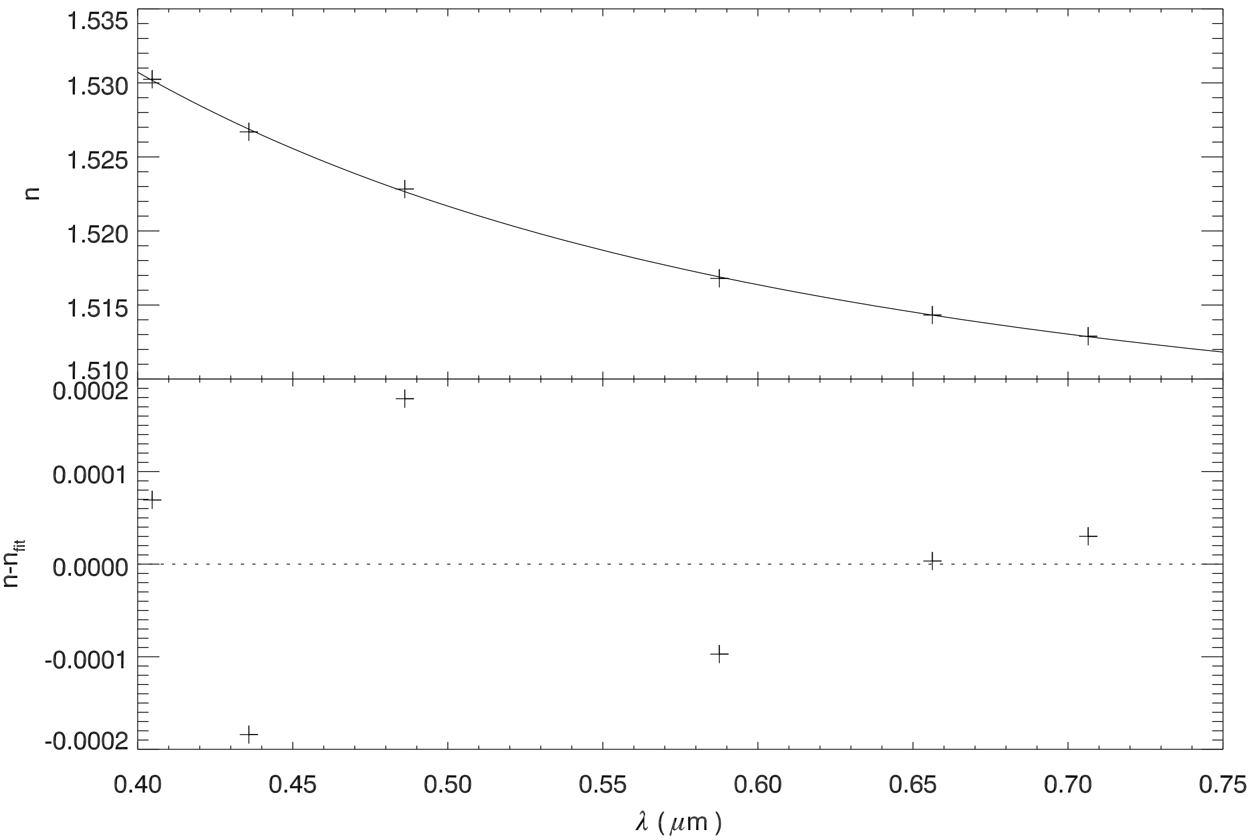
I am going to fit only one of the equations, the Cauchy expression, and only to the λ^{-4} term,

$$n_{\mathrm{Cauchy}}(\lambda) = a + \frac{b}{\lambda^2} + \frac{d}{\lambda^4}$$

I am going to fit the curve tl all of these points. This table lists the wavelengths, the given index of refraction, and the fitted index of refraction.

Line	λ	n	fit
h	0.40466	1.53024	1.53017
g	0.43584	1.52669	1.52687
F	0.48613	1.52283	1.52265
d	0.58756	1.51680	1.51670
C	0.65627	1.51432	1.51432
r	0.70652	1.51289	1.51286

The following plot shows the data and the model plotted together int he top panel, and the fit residuals in the lower panel. The RMS variation from the model is 0.00012.



Problem 4

This is the product of the two curves. Sorry, but I don't feel like measuring the curves, and I don't have to because I am the teacher.

Problem 5

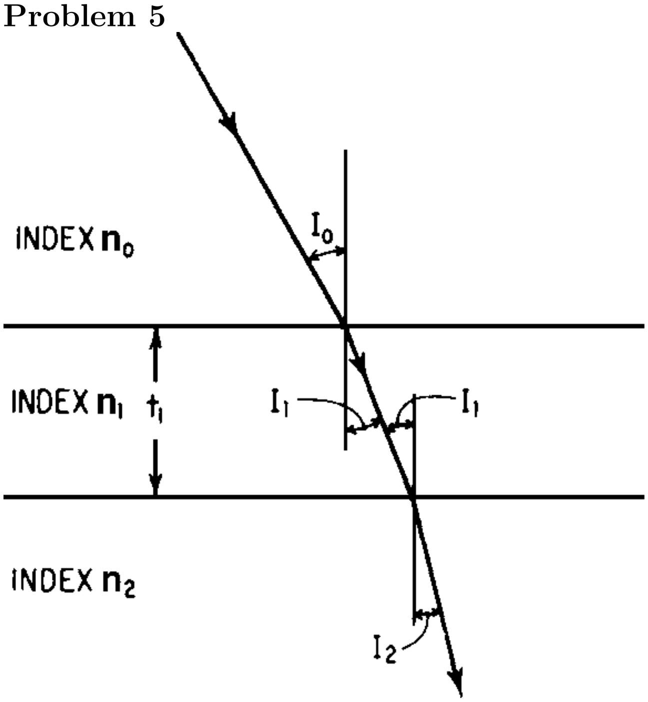


Figure 7.15 Passage of light ray through a thin film, indicating the terms used in Eq. 7.21.

We need to use equation 7.21,

$$R = \frac{r_1^2 + r_2^2 + 2r_1 r_2 \cos X}{1 + r_1^2 + r_2^2 + 2r_1 r_2 \cos X},$$

where

$$r_1 = -\frac{\sin(I_0 - I_1)}{\sin(I_0 + I_1)} \text{ or } r_1 = \frac{\tan(I_0 - I_1)}{\tan(I_0 + I_1)}$$

and

$$r_2 = -\frac{\sin(I_1 - I_2)}{\sin(I_1 + I_2)} \text{ or } r_2 = \frac{\tan(I_1 - I_2)}{\tan(I_1 + I_2)}$$

and

$$X = \frac{4\pi n_1 t_1 \cos I_1}{\lambda}$$

I_0, I_1, and I_2 are related by Snell's law,

$$n_0 \sin I_0 = n_1 \sin I_1 \qquad n_1 \sin I_1 = n_2 \sin I_2$$

The indices are $n_0 = 1$, $n_1 = 1.38$, and $n_2 = 1.52$. The following plot shows the reflectivity for s-polarized light (solid) and p-polarized light (dotted).