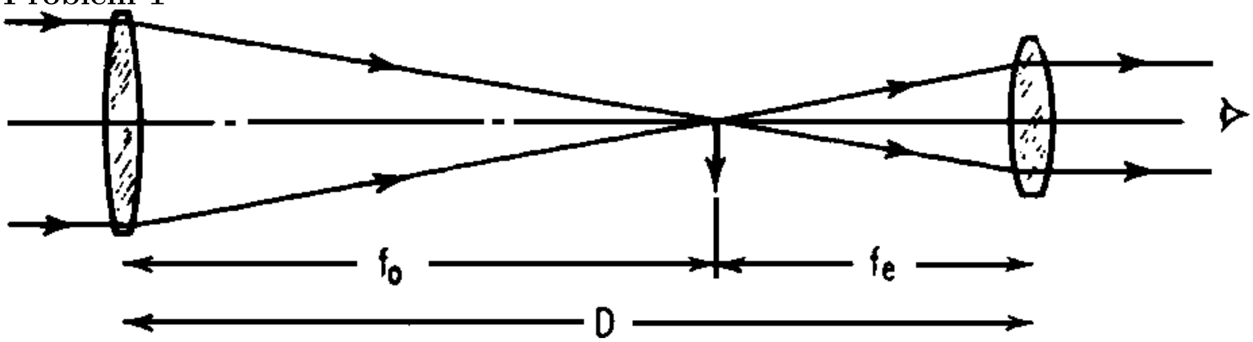
Solutions to homework #9 due 2007/4/24

Problem 1



(a) We know that $D = 10\,\text{in}$, and that $f_o + f_e = D$. If the magnification is $M = 20\times$, then a parallel ray must enter the objective lens 20 times further away from the optical axis than it exits at the eye lens, so $y_{\text{objective}} = My_{\text{eye}}$. If we consider similar triangles, then it must also be true that

$$\frac{y_{\text{objective}}}{y_{\text{eye}}} = \frac{f_o}{f_e}$$

or

$$M = \frac{f_o}{f_e}$$

We now have two equations with two unknowns:

$$f_o + f_e = D \qquad\qquad M = \frac{f_o}{f_e}$$

The solution is

$$f_o = \frac{MD}{M+1} = \frac{20 \times 10}{21} = 9.52\,\text{in}$$

and

$$f_e = \frac{D}{M+1} = \frac{10}{21} = 0.48\,\text{in}$$

(b) The eye relief is the distance from the last optical surface to the exit pupil. Since we assume thin lenses, it is the distance from the seond lens to the exit pupil. The exit pupil is the image of the aperture, which we assume to be the objective in this case. We use the well-known formula

$$\frac{1}{s'} = \frac{1}{s} + \frac{1}{f}$$

where $s = -10\,\text{in}$, and $f = 0.48\,\text{in}$. So we get

$$s' = \left(-\frac{1}{10} + \frac{1}{0.48}\right)^{-1} = 0.50\,\text{in}$$

(c) The resolution of the eye is $1'$. Since the telescope provides $20\times$ magnification, the diffraction-limited resolution of the objective lens must be $\frac{1}{20}'$. We then use the Rayleigh criterion, and assume a wavelength of 500 nm. We get

$$1.22\frac{\lambda}{D} = \theta \Rightarrow D = 1.22\frac{\lambda}{\theta}$$

Inserting we get $D = 4.19\,\mathrm{cm} = 1.65\,\mathrm{in}$

(d) I don't know what he means by real field, but I am going to calculate the unvignetted field. That is the entrance angle of a ray entering at the top of the objective and passing the bottom of the eye lens. I assume that the diameter of the objective is 1.83 in. In that case the angle of that ray is

$$u_1 = \frac{-0.25 - \frac{1.83}{2}}{10} = -0.1165$$

The angle of the entering ray can be obtained through ray tracing through the objective lens.

$$u_1 = u_0 - \frac{y_0}{f}$$

so

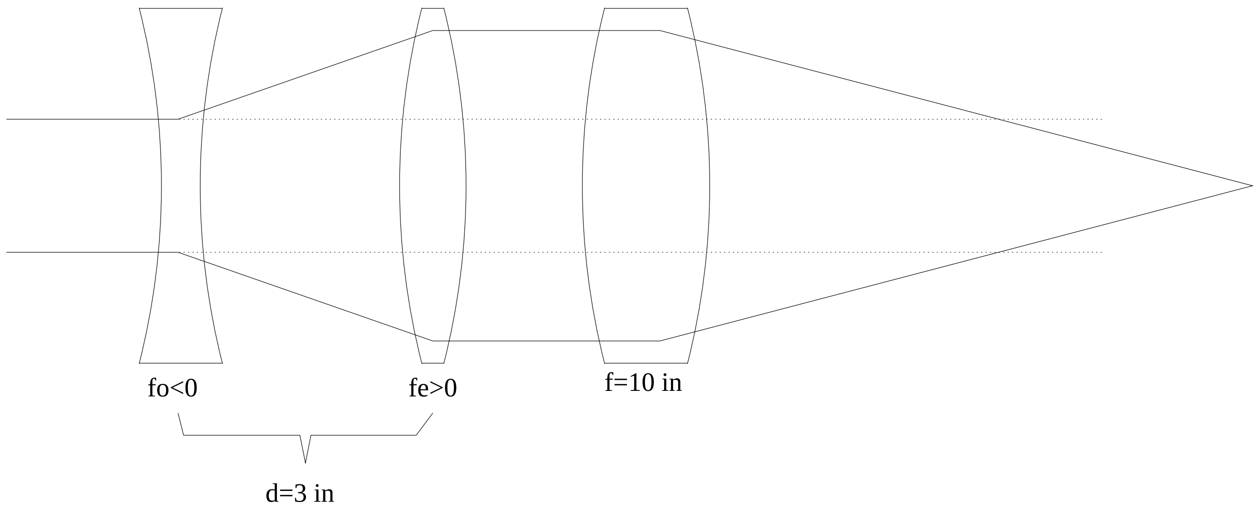
$$u_0 = u_1 + \frac{y_0}{f} = -0.1165 + \frac{\frac{1.83}{2}}{9.52} = -0.0204$$

The un-vignetted field of view is therefore $\pm 0.0204 = 1.17°$. The 100% vignetted field of view corresponds to the ray which goes from the bottom of the objective lens to the bottom of the eye lens. It travels a vertical distance of $1.83/2 - 0.25 = 0.66$, so it has an angle of $u_1 = 0.0665$. Converting that to entrance angle in the same way as above we get

$$u_0 = u_1 + \frac{y_0}{f} = 0.0665 - \frac{\frac{1.83}{2}}{9.52} = -0.0296$$

The fully vignetted field of view is therefore $\pm 0.0296 = 1.70°$.

Problem 2
This system is very similar to the one displayed in Figure 9.26 on page 288, except that the discussion in Figure 9.26 pertained to cylindrical elements. But conventional spherical elements are very similar.



(a) The effective focal length, efl is defined as y_1/u_k', where y_1 is the entrance height of a ray, and u_k' is the exit angle of the ray. The afocal telescope does not change the entrance angle of rays, so to reduce the efl by a factor of two, the afocal attachment must increase the entrance beam diameter by a factor of two. This corresponds to a magnification of $M = 0.5$. There are several ways of finding the focal lengths of the two lenses of the telescope attachment. I will use ray-tracing. A ray which enters the first lens at $(y_1, u_1 = 0)$ will exit the second lens at $(2y_1, u_2' = 0)$. The lenses are separated by distance d, so the angle of the rays between the lenses is $u_1' = u_2 = \frac{y_1}{d}$. The ray-tracing equation is

$$u' = u - \frac{y}{f}$$

for the two lense we then have

$$u_1' = u_2 = u_1 - \frac{y_1}{f_1} \qquad u_2' = u_2 - \frac{2y_1}{f_2}$$

Now inserting everything that we know

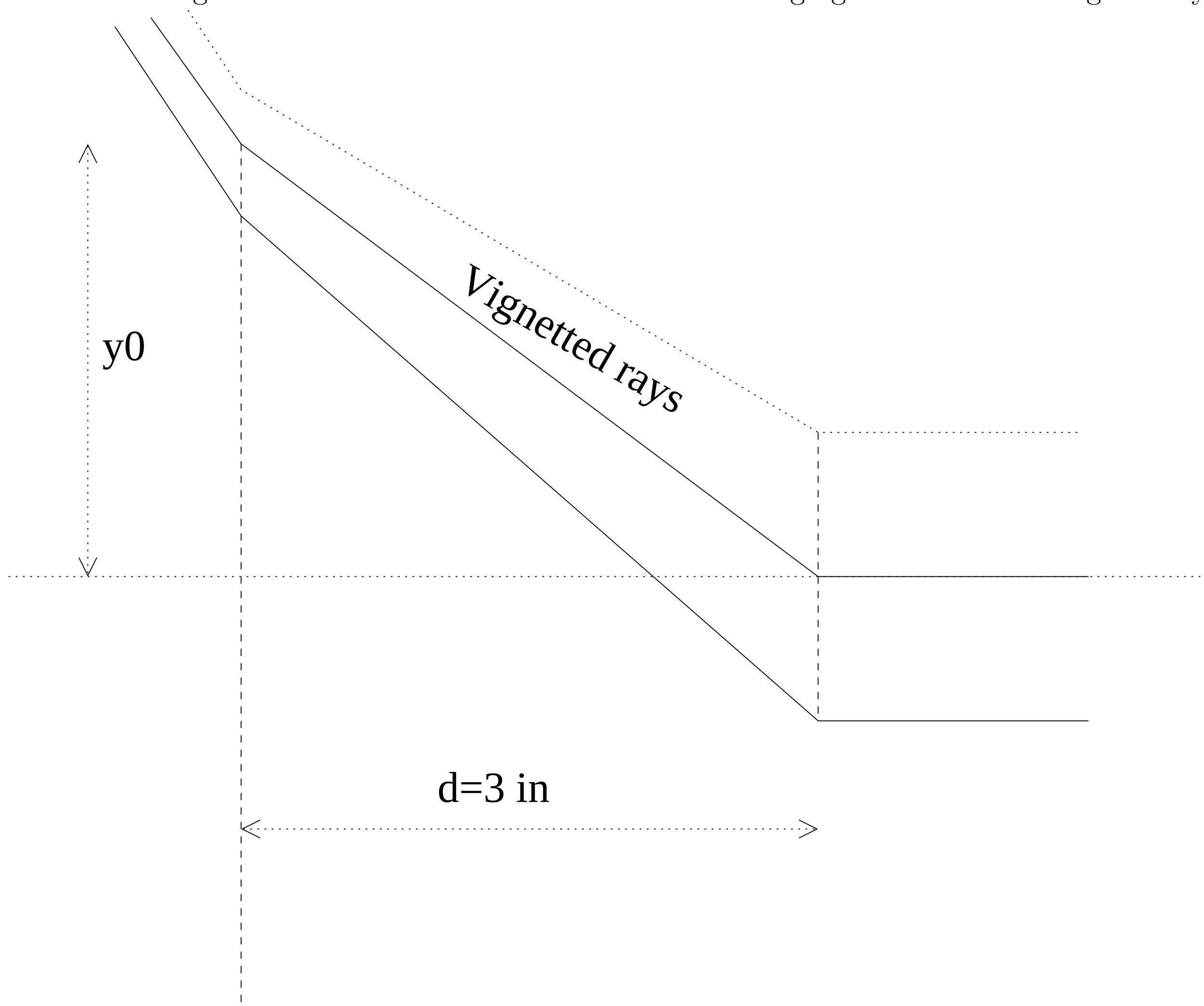
$$\frac{y_1}{d} = -\frac{y_1}{f_1} \qquad 0 = \frac{y_1}{d} - \frac{2y_1}{f_2}$$

which reduces to

$$f_1 = -d \qquad \frac{1}{d} = \frac{2}{f_2} \Rightarrow f_2 = 2d$$

Inserting numbers we get $f_1 = -3\,\text{in}$ and $f_2 = 6\,\text{in}$.
(b) I am going to assume that the second lens is immediately adjacent to the third lens, and that it has the same diameter as the third lens. In that case only need to consider whether

rays entering the first lens will make it through the second lens. For 50% vignetting I am looking at a marginal ray through the first lens which enters the second lens through its center. The question then becomes "What should be the height of that ray through the first lens for the angle into the first lens to be 60°?" The following figure illustrates the geometry



The angle of the ray in the space between the two lenses is thus

$$u_1' = u_2 = -\frac{y_1}{d}$$

and its relationship to the entrance ray angle is

$$u_1' = u_2 = u_1 - \frac{y_1}{f_1}$$

Eliminating u_1' we get

$$-\frac{y_1}{d} = u_1 - \frac{y_1}{f_1}$$

Isolate y_1 we get

$$y_1\left(\frac{1}{f_1}-\frac{1}{d}\right)=u_1 \Rightarrow y_1=\frac{u_1}{\frac{1}{f_1}-\frac{1}{d}}$$

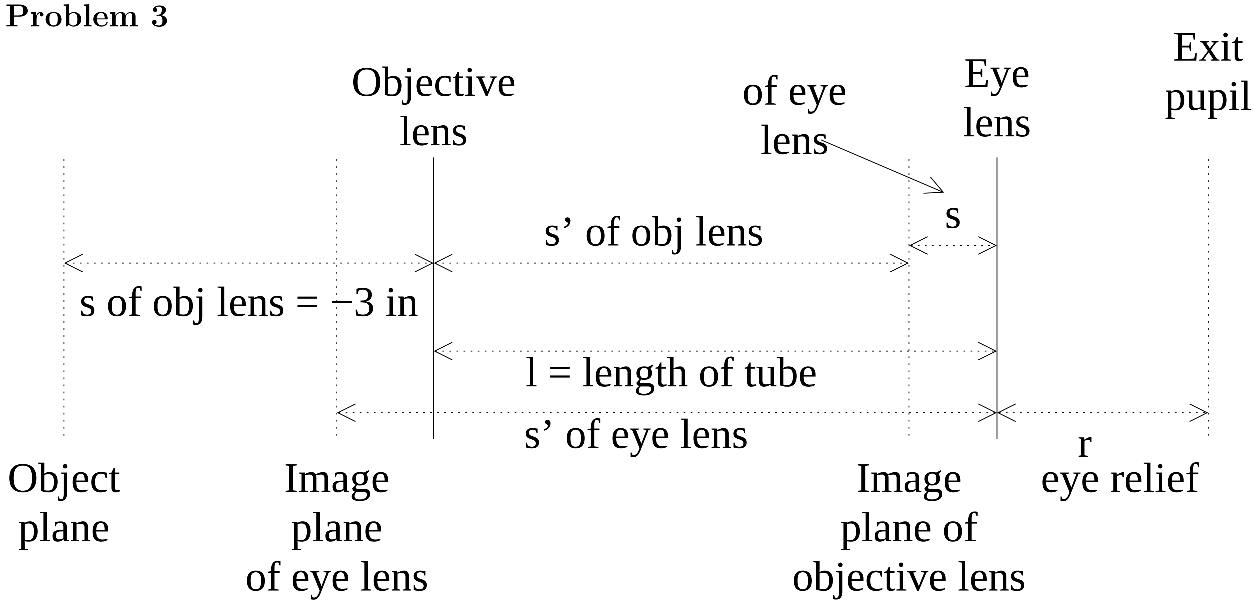
If we set $u_1=-60^\circ=-1.047$, and $f_1=-3\,\text{in}$, and $d=3\,\text{in}$, we can find y_1 as

$$y_1=\frac{-1.047}{-\frac{1}{3}-\frac{1}{3}}=1.571$$

The diameter of the front lens should then be

$$D=2\times 1.571=3.141\,\text{in}$$

Problem 3



(a) The length of the microscope is $l=s'_o-s_e$. We have

$$\frac{1}{s'_o}=\frac{1}{s_o}+\frac{1}{f_o}$$

Inserting $s_o=-3\,\text{in}$ and $f_o=2\,\text{in}$ we get $s'_o=6\,\text{in}$. Next we have

$$\frac{1}{s'_e}=\frac{1}{s_e}+\frac{1}{f_e}$$

Isolating s_e and inserting $s'_e=-\infty$ and $f_e=2\,\text{in}$ we get $s_e=-2\,\text{in}$. The length of the microscope is then $l=s'_o-s_e=8\,\text{in}$.

(b) The magnification power of a microscope can be calculated as the standard viewing distance (10 in) divided by the effective focal length of the microscope,

$$\text{MP}=\left|\frac{10,\text{in}}{f_m}\right|$$

The effective focal length of the microscope can be calculated with the two lens equation

$$f_\text{m}=\frac{f_of_e}{f_o+f_e-l}$$

Inserting $f_o = f_e = 2\,\text{in}$ and $l = 8\,\text{in}$ we get $f_{\text{m}} = -1\,\text{in}$, and $\text{MP} = 10$.

Problem 4

The situation described is one in which the focal points of the lenses do not coincide. The image projected from the first lens is located relative to the focal point of the second lens in such a way that the second lens project a virtual image 20 inches to the left. We know f, s', and we want to find s in the formula.

$$\frac{1}{s'} = \frac{1}{s} + \frac{1}{f}$$

we get

$$s = \left(\frac{1}{s'} - \frac{1}{f}\right)^{-1} = \left(\frac{1}{-20} - \frac{1}{5}\right)^{-1} = -4$$

The two lense are thus separated by $9\,\text{in}$. The magnification is the ratio of beam diameters. The beam diameter varies with location of the eye. Rays from an infinitely distant object will strike the eye lens at $4/5$ of its diameter. The magnification is thus $5/4$, of 1.25. The pupil is the image of the objective lens, and its location, s', can be found from

$$\frac{1}{s'} = \frac{1}{s} + \frac{1}{f}$$

where $s = -9\,\text{in}$, and $f = 5\,\text{in}$. We get $s' = 11.25\,\text{in}$. At the pupil, rays from an object point $20\,\text{in}$ behind the lens will have expanded to $4/5 \times \frac{31.25}{20} = 5/4$. The magnification is thus $4/5 = 0.8$.

Problem 5

The visual acuity for rangefinders is 10". If we include a $20\times$ telescope, then the precision of alignment is 0.5". The relationship between baseline, B, angle, θ, and distance, D is

$$\frac{B}{D} = \theta \Rightarrow D = \frac{B}{\theta}$$

Assuming a perfectly determined baseline, the relationship between distance uncertainty and angle uncertainty is

$$\sigma_D^2 = \left(\frac{B}{\theta^2}\right)^2 \sigma_\theta^2$$

We want to determine the B which will produce $\frac{\sigma_D}{D} = 0.5\%$. Eliminate θ.

$$\sigma_D^2 = \left(\frac{BD^2}{B^2}\right)^2 \sigma_\theta^2$$

or

$$\sigma_D = \frac{D^2}{B}\sigma_\theta$$

or

$$\frac{\sigma_D}{D} = \frac{D}{B}\sigma_\theta$$

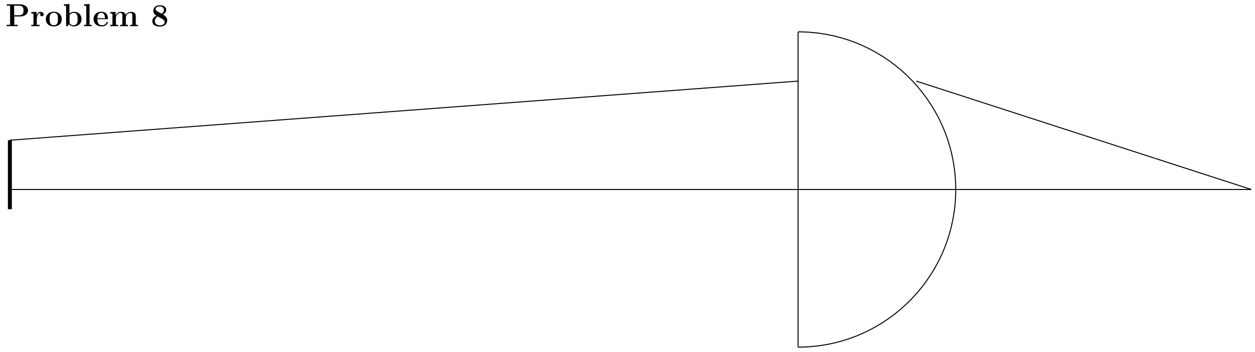
Now we isolate B,

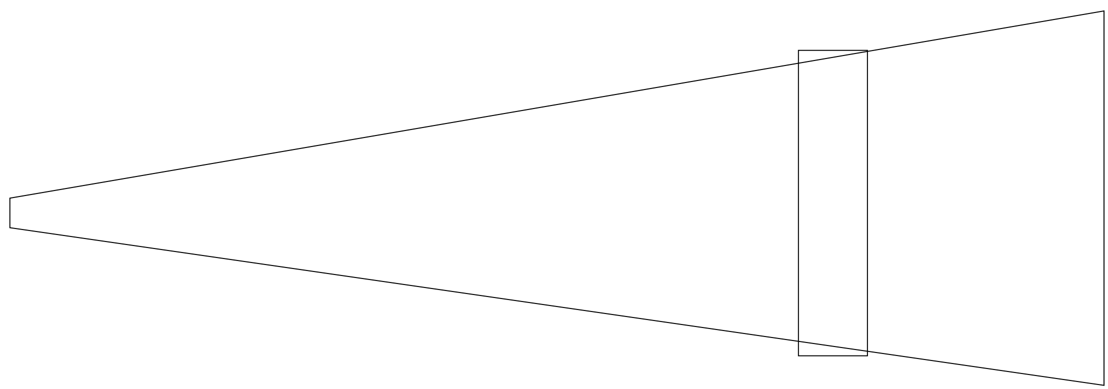
$$B = \frac{D}{\frac{\sigma_D}{D}} \sigma_\theta$$

Now insert values

$$B = \frac{2000}{0.5 \times 10^{-2}} 0.5\text{"} = 0.97\,\mathrm{m}$$

Problem 8





First, let's find the focal length of the lens,

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{t\,(n-1)}{R_1 R_2 n}\right)$$

Using $R_1 = \infty$, $R_2 = -2.5\,\mathrm{mm}$, $t = 2.5\,\mathrm{mm}$, and $n = 1.5$, we get

$$f = 5.0\,\mathrm{mm}$$

Next we want to know image location and magnification. We already know that $s = -50\,\mathrm{mm}$ (1st principal point is at plane surface). We want to know s'

$$\frac{1}{s'} = \frac{1}{s} + \frac{1}{f}$$

We get $s' = 5.56\,\mathrm{mm}$. Now we can compute the magnification,

$$m = \frac{s'}{s} = -\frac{5.56}{50} = 0.11$$

In the opposite direction, the 20 mm wide beam of light will keep expanding until the focus. At a distance of 50 mm it is 20 mm wide, and at a distance of 55.56 mm it is 20 mm $\times$ $\frac{55.56}{50} = 22.22$ mm wide.

Problem 9

This problem refers to the figure and expressions on page 294. There is a factor of 4 zoom ratio. That can mean that $M = 4$ or $M = 0.25$.

At minimum shift, $s_1 + s_2 = 10$ in, and at minimum shift $-l' = 10$ in.

We get

$$\Phi_A = \frac{R-1}{R\,(s_1+s_2)}$$

$$\Phi_B = -\Phi_A\,(R+1)$$

$$\Phi_C = \frac{(\Phi_A+\Phi)\,R\,(R+1)}{3R-1}$$

where $R = \sqrt{M}$. For $M = 4$, these expressions evaluate to

$$\Phi_A = 0.05\,\text{in}^{-1} \qquad \Phi_B = -0.15\,\text{in}^{-1} \qquad \Phi_C = 0.18\,\text{in}^{-1}$$

and for $M = 0.25$, they evaluate to

$$\Phi_A = -0.1\,\text{in}^{-1} \qquad \Phi_B = 0.15\,\text{in}^{-1} \qquad \Phi_C = 0$$

The elements separations at minimum shift are

$$s_1 = \frac{R-1}{\Phi_A\,(R+1)}$$

$$s_2 = \frac{R-1}{\Phi_A R\,(R+1)}$$

For $M = 4$, these expressions evaluate to

$$s_1 = 6.67\,\text{in} \qquad s_2 = 3.33\,\text{in}$$

and for $M = 0.25$ we get

$$s_1 = 3.33\,\text{in} \qquad s_2 = 6.67\,\text{in}$$

The back focal length, l', is

$$l' = \frac{3R-1}{\Phi R\,(R+1)}$$

and at minimum shift, $\Phi = 0.1\,\text{in}^{-1}$, so we get $l' = 8.33$ in and $l' = 6.67$ in for $M = 4$ and $M = 0.25$ respectively.