Lecture 1

Measurement Systems

EE 521: Instrumentation and Measurements

Lecture Notes from August 30, 2009

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Contents

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1 Course Overview

Textbooks

- C.W. De Silva, *Sensors and Actuators: Control System Instrumentation*, CRC Press, 2007.
- R. Northrop, *Introduction to Instrumentation and Measurements*, 2nd Edition, CRC Press, 2005.

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Topics Covered

- Measurement systems
- Error Analysis
- Signal conditioning
- Noise and interference
- Survey of different types of sensor input mechanisms
- Sensor Applications
- Data acquisition and digital signal processing

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Grading

- Homework: 30%
- Final: 20%
- Project and Research Paper: 20%
- Presentations: 20%
- Class Participation: 10%

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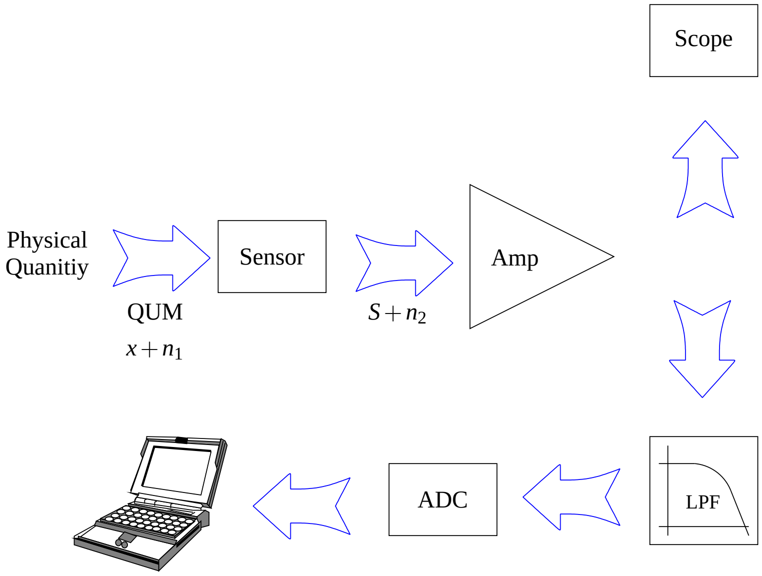
2 Measurement System



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3 Noise

Noise Sources

1. Environment noise: associated with the physical quantity being measured.
2. Measurement noise: due to the measurement system.
3. Quantization noise: due to digitizing the analog signal. 1.7

Error in Measurement

1. Gross Errors
   - Taking measurement during transient time of the instrument.
   - Mistakes recording data or calculating a derived measurand.
   - Misuse of the instrument.
2. System Errors
   - Calibration errors.
   - Random noise both internal and external.
   - Drift. 1.8

4 Error Analysis

Error

It is important to find a way to describe errors in measurements in terms of some accepted concepts. For example, accuracy, precision, limiting error and error statistics. 1.9

Accuracy

Error

$$\varepsilon_n \equiv X_n - Y_n \tag{1}$$

and

$$\%\varepsilon \equiv \left| \frac{\varepsilon_n}{Y_n} \right| \times 100 \tag{2}$$

where Y_n is the true value and X_n is the actual measurement.

Dilemma
To compute the error we need to know the true value. If the true value is known, why measure it?

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Accuracy

$$A_n \equiv 1 - \left| \frac{Y_n - X_n}{Y_n} \right| \tag{3}$$

or

$$\%A_n \equiv 100 - \%\varepsilon \tag{4}$$

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Precision

$$P_n \equiv 1 - \left| \frac{X_n - \overline{X}}{X} \right| \tag{5}$$

where

$$\overline{X} \equiv \frac{1}{N} \sum_{n=1}^{N} X_n \tag{6}$$

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Deviation

- Deviation

$$d_n \equiv X_n - \overline{X} \tag{7}$$

- Average Deviation

$$D_N = \frac{1}{N} \sum_{n=1}^{N} d_n \tag{8}$$

- Standard Deviation

$$S_N = \frac{1}{N} \sum_{n=1}^{N} d_n^2 = \sigma_x \tag{9}$$

- Variance

$$S_N^2 = \frac{1}{N} \sum_{n=1}^{N} X_n^2 - (\overline{X})^2 = \sigma_x^2 \tag{10}$$

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Limiting Error (LE)

A term frequently used by manufacturers to describe worst case error

Taylor's Series

$$f(X \pm \Delta X) = f(X) + \frac{df}{dX}\frac{\Delta X}{1!} + \cdots + \frac{d^{n-1}f}{dX^{n-1}}\frac{(\Delta X)^{n-1}}{(n-1)!}$$

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Limiting Error (LE)

Assuming that QUM is a function of N variables

$$Q = f(X_1, X_2, \ldots, X_N) \tag{11}$$

but due to measurement errors

$$\hat{Q} = f(X_1 \pm \Delta X_1, X_2 \pm \Delta X_2, \ldots, X_n \pm \Delta X_N) \tag{12}$$

$$\hat{Q} = f(X_1, X_2, \ldots, X_N) + \left\{ \frac{\partial f}{\partial X_1}\Delta X_1 + \frac{\partial f}{\partial X_2}\Delta X_2 + \cdots + \frac{\partial f}{\partial X_N}\Delta X_N \right\} + \cdots$$
$$\frac{1}{2!}\left\{ \frac{\partial^2 f}{\partial X_1^2}(\Delta X_1)^2 + \frac{\partial^2 f}{\partial X_2^2}(\Delta X_2)^2 + \cdots + \frac{\partial^2 f}{\partial X_N^2}(\Delta X_N)^2 \right\} + \cdots$$

Using Taylor series and ignoring the higher order terms.

$$\Delta Q_{MAX} = |Q - \hat{Q}| = \sum_{j=1}^{N} \left| \frac{\partial f}{\partial X_j}\Delta X_j \right| \tag{13}$$

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Linear Regression - Least Mean Square Linear Fitting

Fit a line with equation $y = mx + b$ to a set of N noisy measurements.

Minimize

$$\sigma_y^2 = \frac{1}{N}\sum_{k=1}^{N}\left[(mX_k + b) - Y_k\right]^2 \tag{14}$$

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Linear Regression - Least Mean Square Linear Fitting

Differentiating with respect to m and equating to zero

$$\frac{\partial \sigma_y^2}{\partial m} = \frac{2}{N}\sum_{k=1}^{N}\left[(mX_k + b) - Y_k\right]X_k = 0 \tag{15}$$

Differentiating with respect to b and equating to zero

$$\frac{\partial \sigma_y^2}{\partial b} = \frac{2}{N}\sum_{k=1}^{N}\left[(mX_k + b) - Y_k\right] = 0 \tag{16}$$

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Linear Regression - Least Mean Square Linear Fitting

Solving the previous two equations of m and b

$$m = \frac{R_{xy}(0) - \overline{Y}\,\overline{X}}{\sigma_x^2} \tag{17}$$

$$b = \frac{\overline{Y}\,\overline{X}^2 - \overline{X}R_{xy}(0)}{\sigma_x^2} \tag{18}$$

where R_{xy} is the cross-correlation of X and Y evaluated at zero

$$R_{xy}(0) = \frac{1}{N}\sum_{k=1}^{N} X_k Y_k \tag{19}$$

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Linear Regression - Goodness of fit

Using the correlation coefficient

$$r \equiv \frac{R_{xy}(0) - \overline{X}\,\overline{Y}}{\sigma_x \sigma_y}, \qquad 0 \le r \le 1 \tag{20}$$

Perfect fit if $r = 1$. 1.19