# Lecture 2 - A Analog Signal Conditioning 

EE 521: Instrumentation and Measurements
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## 2 Differential Amplifiers (DA)

Overview

- Used for a wide variety of applications.
- Responds to the use difference in the input signals.
- Discriminates against changes in the dc power supply voltage.


## DA Basics

Given two inputs $v_{i, 1}$ and $v_{i, 2}$ define difference mode and common-mode input and output signals as

$$
\begin{array}{cl}
v_{i d}=\left(v_{i, 1}-v_{i, 2}\right) / 2, & v_{i c}=\left(v_{i, 1}+v_{i, 2}\right) / 2 \\
v_{o d}=\left(v_{o, 1}-v_{o, 2}\right) / 2, & v_{o c}=\left(v_{o, 1}+v_{o, 2}\right) / 2 \tag{2}
\end{array}
$$

See Figure 1.


Figure 1: Differential amplifier block

$$
\begin{equation*}
v_{o d}=A_{d d} v_{i d}+A_{d c} v_{i c}, \quad v_{o c}=A_{c d} v_{i d}+A_{c c} v_{i c} \tag{3}
\end{equation*}
$$

## Output Equations

For single output $v_{o}=v_{o, 1}$

$$
\begin{align*}
v_{o} & =\frac{A_{d d}+A_{c d}+A_{d c}+A_{c c}}{2} v_{i, 1}+\frac{A_{c c}+A_{d c}-A_{d d}-A_{c d}}{2} v_{i, 2}  \tag{4}\\
& =A_{D} v_{i d}+A_{C} v_{i c}
\end{align*}
$$

Note. These gains are frequency dependent.

## Ideal DA

$$
\begin{align*}
A_{c c}=A_{c d}=A_{d c} & =0  \tag{5}\\
A_{C}=0, \quad A_{D} & =A_{d d} \tag{6}
\end{align*}
$$

## Common-Mode Rejection Ratio (CMRR)

How close is the real differential amplifier close to an ideal one?

$$
\begin{gather*}
C M R R \equiv \frac{\left(\left|v_{i c}\right| \text { to give a certain } v_{o}\right)}{\left(\left|v_{i d}\right| \text { to give the same } v_{o}\right)}  \tag{7}\\
C M R R=\left|\frac{A_{d d}}{A_{c c}}\right|=\left|\frac{A_{D}}{A_{C}}\right| \tag{8}
\end{gather*}
$$

### 2.1 CMRR - Measurement

Measurement of $A_{D}$ and $A_{C}$

- First case, connect both inputs to $v_{s 1}$ generating a common mode input signal which results in an output $v_{o}$.
- Second Case, connect the $+\operatorname{input}$ to $v_{s 2}$ and negative input to ground.

$$
v_{i d}=v_{s 2} / 2, \quad v_{i c}=v_{s 2} / 2
$$

- Adjust $v_{s 2}$ such that the output is equal to $v_{o}$ generated in the common mode input.

$$
A_{C}=v_{o} / v_{i c}=v_{o} / v_{s 1}
$$

also

$$
v_{o}=A_{D} v_{s 2} / 2+A_{C} v_{s 2} / 2
$$

consequently,

$$
\begin{equation*}
A_{D}=v_{o}\left(2 / v_{s 2}-1 / v_{s 1}\right) \tag{9}
\end{equation*}
$$

and

$$
\begin{equation*}
C M R R=A_{D} / A_{C}=\left(2 v_{s 1} / v_{s 2}-1\right) \tag{10}
\end{equation*}
$$

### 2.2 Source Resistance Asymmetry

## DC Equivalent Input Circuit

Most frequently, common-mode input resistance, $R_{i c}$, measured between one input to ground using common mode input, and a difference-mode input resistance, $R_{i d}$, measured using difference mode input from either inputs to ground, are provided by the manufacturer.


Figure 2: Differential amplifier input circuit at dc showing source resistance unbalance.

## DA Input Circuit

See Figure 2

## CMRR-Computation

Under difference mode excitation, the current into the non-inverting input is given by:

$$
\begin{equation*}
i_{d}=2 v_{i d} / R_{1}+v_{i d} / R_{i c} \tag{11}
\end{equation*}
$$

and

$$
\begin{gather*}
i_{d} / v_{i d}=1 / R_{i d}=2 / R_{1}+1 / R_{i c}  \tag{12}\\
R_{1}=2 R_{i d} R_{i c} /\left(R_{i c}-R_{i d}\right) \tag{13}
\end{gather*}
$$

If $R_{i c}=R_{i d}$, then $R_{1}=\infty$. Using this assumption as well as the fact that $\delta R$ is numerically negligible, then under common mode excitation $v_{s c}$,

$$
\begin{gather*}
\frac{v_{i d}}{v_{s c}}=\frac{R_{i c} \delta R}{2\left(R_{i c}+R_{s}\right)^{2}}  \tag{14}\\
\frac{v_{i c}}{v_{s c}}=\frac{R_{i c}}{R_{i c}+R_{s}} \tag{15}
\end{gather*}
$$

For purely difference mode excitation $v_{s d}$

$$
\begin{gather*}
\frac{v_{i d}}{v_{s d}}=\frac{R_{i c}}{R_{i c}+R_{s}}  \tag{16}\\
\frac{v_{i c}}{v_{s d}}=\frac{R_{i c} \delta R}{2\left(R_{i c}+R_{s}\right)^{2}} \tag{17}
\end{gather*}
$$

Using $v_{o}=A_{D} v_{i d}+A_{C} v_{i c}$,

$$
\begin{equation*}
v_{o}=A_{D} v_{s c} R_{i c} \delta R /\left[2\left(R_{i c}+R_{s}\right)^{2}\right]+A_{C} v_{s c} R_{i c} /\left(R_{i c}+R_{s}\right) \tag{18}
\end{equation*}
$$

and

$$
\begin{gather*}
v_{o}=A_{D} v_{s d} R_{i c} /\left(R_{i c}+R_{s}\right)+A_{C} v_{s d} R_{i c} \delta R /\left[2\left(R_{i c}+R_{s}\right)^{2}\right]  \tag{19}\\
C M R R_{s y s}=\frac{A_{D} / A_{C}+\delta R /\left[2\left(R_{i c}+R_{s}\right)\right]}{\left[A_{D} / A_{C}\right] \delta R /\left[2\left(R_{i c}+R_{s}\right)\right]+1} \\
\approx \frac{C M R R_{A}}{C M R R_{A} \delta R /\left[2\left(R_{i c}+R_{s}\right)\right]+1} \tag{20}
\end{gather*}
$$

$C M R R_{A}$ is specified by the manufacturer


Figure 3: Differential amplifier CMRR magnitude vs fractional unbalance in source resistance.

## CMRR - Special Cases

- If the thevenin source resistance are matched

$$
\begin{equation*}
C M R R_{s y s}=C M R R_{A} \tag{21}
\end{equation*}
$$

- When

$$
\delta R / R_{s}=-2\left(R_{i c} / R_{s}+1\right) / C M R R_{A}
$$

then

$$
C M R R_{\text {sys }} \rightarrow \infty
$$

## Source Resistance Asymmetry

See Figure 3 We can increase the systems CMRR by adding an external resistance.

## 3 Operational Amplifiers

## Typical Op-Amp

Open-loop Transfer Function

$$
\begin{equation*}
A_{D}=\frac{v_{o}}{v_{i}-v_{i}^{\prime}}=\frac{k_{o v}}{\left(\tau_{1} s+1\right)\left(\tau_{2} s+1\right)} \tag{22}
\end{equation*}
$$

- $f_{1}=1 /\left(2 \pi \tau_{1}\right)$ and $f_{2}=1 /\left(2 \pi \tau_{2}\right)$.
- To ensure stability $\left|A_{D}\left(j f_{2}\right)\right| \ll 1$.
- $f_{1}$ occurs at a relatively low value.
- Small-signal gain $\times$ bandwidth product

$$
\begin{equation*}
G B W P \approx k_{o v} /\left(2 \pi \tau_{1}\right) \tag{23}
\end{equation*}
$$

- The unity gain of 0 dB frequency of the open loop, $f_{T}$, is approximately equal to the $G B W P$.
- Slew rate $\eta$.


## Notes

- Two pieces of information define are required to define the op-amp's open-loop characteristics.
- DC open-loop gain $k_{o v}$, and
- 0 dB frequency, $f_{T}$.
- Closed-loop gain is limited by the open-loop gain.
- The higher the closed-loop gain is the smaller the bandwidth.
- To overcome this problem, it is better to cascade identical amplification stages rather than having one op-amp with a high gain.


### 3.1 Broadband Amplifier

Non-inverting Amplifier
See Figure 4A.

$$
\begin{gather*}
v_{o}=\left(v_{s}-v_{i}^{\prime}\right) \frac{k_{v o}}{\tau s+1}  \tag{24}\\
v_{i}^{\prime}=v_{o} \frac{R_{1}}{R_{1}+R_{F}}=\beta v_{o} \tag{25}
\end{gather*}
$$

(A)
(B)


Figure 4: Non-inverting op-amp circuits

$$
\begin{equation*}
v_{o} / v_{s}=\frac{k_{o v} /(\tau s+1)}{1+\beta k_{o v} /(\tau s+1)} \tag{26}
\end{equation*}
$$

which, assuming that $\beta k_{o v} \gg 1$ reduces to

$$
\begin{gather*}
v_{o} / v_{s}=\frac{1 / \beta}{s\left(\tau /\left(\beta k_{o v}\right)\right)+1}  \tag{27}\\
G B W P=\left[k_{o v} /\left(1+\beta k_{o v}\right)\right]\left[\left(\beta k_{o v}\right) /(2 \pi \tau)\right]=k_{o v} /(2 \pi \tau) \tag{28}
\end{gather*}
$$

If $R_{1}$ is infinite and $R_{F}$ is shortcircuit, then we obtain a voltage follower that can be used as impedance isolation, see Figure 4B.

Inverting Amplifier and Summer
See Figure 5. The gain for the $k$ th inverting input is

$$
\begin{equation*}
v_{o} / v_{s k}=\frac{-\left(G_{1} / G_{F}\right)}{s\left(\tau \sum G /\left(k_{o v} G_{F}\right)+1\right.} \tag{29}
\end{equation*}
$$



Figure 5: Summing op-amp

