# Lecture 2 - A Analog Signal Conditioning

EE 521: Instrumentation and Measurements

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Aly El-Osery, Electrical Engineering Dept., New Mexico Tech

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# Contents

1	Op-Amps - Handbook	1
2	Differential Amplifiers (DA)	1
	2.1 CMRR - Measurement	2
	2.2 Source Resistance Asymmetry	2
3	Operational Amplifiers	4
	3.1 Broadband Amplifier	5

# 1 Op-Amps - Handbook

Handbook of Operational Amplifier http://focus.ti.com/lit/an/sboa092a/sboa092a.pdf

# 2 Differential Amplifiers (DA)

## Overview

- Used for a wide variety of applications.
- Responds to the use difference in the input signals.
- Discriminates against changes in the dc power supply voltage.

### **DA Basics**

Given two inputs  $v_{i,1}$  and  $v_{i,2}$  define *difference mode* and *common-mode* input and output signals as

$$v_{id} = (v_{i,1} - v_{i,2})/2, \qquad v_{ic} = (v_{i,1} + v_{i,2})/2$$
(1)

$$v_{od} = (v_{o,1} - v_{o,2})/2, \qquad v_{oc} = (v_{o,1} + v_{o,2})/2$$
 (2)

See Figure 1.

 $v_{i,1}$   $v_{o,1}$   $v_{o,1}$  $v_{i,2}$   $v_{o,2}$ 

Figure 1: Differential amplifier block

**Middlebrook Equations** 

$$v_{od} = A_{dd}v_{id} + A_{dc}v_{ic}, \qquad v_{oc} = A_{cd}v_{id} + A_{cc}v_{ic}$$

**Output Equations** 

For single output  $v_o = v_{o,1}$ 

$$v_{o} = \frac{A_{dd} + A_{cd} + A_{dc} + A_{cc}}{2} v_{i,1} + \frac{A_{cc} + A_{dc} - A_{dd} - A_{cd}}{2} v_{i,2}$$

$$= A_{D} v_{id} + A_{C} v_{ic}$$
(4)

Note. These gains are frequency dependent.

**Ideal DA** 

$$A_{cc} = A_{cd} = A_{dc} = 0 \tag{5}$$

$$A_C = 0, \qquad A_D = A_{dd} \tag{6}$$

#### Common-Mode Rejection Ratio (CMRR)

How close is the real differential amplifier close to an ideal one?

$$CMRR \equiv \frac{(|v_{ic}| \text{ to give a certain } v_o)}{(|v_{id}| \text{ to give the same } v_o)}$$
(7)

$$CMRR = \left| \frac{A_{dd}}{A_{cc}} \right| = \left| \frac{A_D}{A_C} \right| \tag{8}$$

# 2.1 CMRR - Measurement

#### Measurement of $A_D$ and $A_C$

- First case, connect both inputs to  $v_{s1}$  generating a common mode input signal which results in an output  $v_o$ .
- Second Case, connect the + input to  $v_{s2}$  and negative input to ground.

$$v_{id} = v_{s2}/2, \quad v_{ic} = v_{s2}/2$$

• Adjust  $v_{s2}$  such that the output is equal to  $v_o$  generated in the common mode input.

$$A_C = v_o / v_{ic} = v_o / v_{s1}$$

also

$$v_o = A_D v_{s2}/2 + A_C v_{s2}/2$$

consequently,

$$A_D = v_o(2/v_{s2} - 1/v_{s1}) \tag{9}$$

and

$$CMRR = A_D / A_C = (2v_{s1} / v_{s2} - 1)$$
(10)

# 2.2 Source Resistance Asymmetry

## DC Equivalent Input Circuit

Most frequently, common-mode input resistance,  $R_{ic}$ , measured between one input to ground using common mode input, and a difference-mode input resistance,  $R_{id}$ , measured using difference mode input from either inputs to ground, are provided by the manufacturer.

(3)

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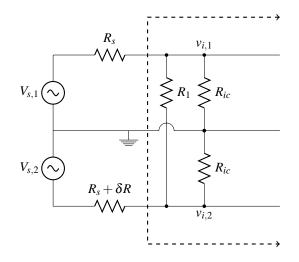


Figure 2: Differential amplifier input circuit at dc showing source resistance unbalance.

# **DA Input Circuit**

See Figure 2

## **CMRR-Computation**

Under difference mode excitation, the current into the non-inverting input is given by:

$$i_d = 2v_{id}/R_1 + v_{id}/R_{ic} \tag{11}$$

and

$$i_d / v_{id} = 1 / R_{id} = 2 / R_1 + 1 / R_{ic} \tag{12}$$

$$R_1 = 2R_{id}R_{ic}/(R_{ic} - R_{id})$$
(13)

If  $R_{ic} = R_{id}$ , then  $R_1 = \infty$ . Using this assumption as well as the fact that  $\delta R$  is numerically negligible, then under common mode excitation  $v_{sc}$ ,

$$\frac{v_{id}}{v_{sc}} = \frac{R_{ic}\delta R}{2(R_{ic} + R_s)^2} \tag{14}$$

$$\frac{v_{ic}}{v_{sc}} = \frac{R_{ic}}{R_{ic} + R_s} \tag{15}$$

For purely difference mode excitation  $v_{sd}$ 

$$\frac{v_{id}}{v_{sd}} = \frac{R_{ic}}{R_{ic} + R_s} \tag{16}$$

$$\frac{v_{ic}}{v_{sd}} = \frac{R_{ic}\delta R}{2(R_{ic} + R_s)^2} \tag{17}$$

Using  $v_o = A_D v_{id} + A_C v_{ic}$ ,

$$v_o = A_D v_{sc} R_{ic} \delta R / [2(R_{ic} + R_s)^2] + A_C v_{sc} R_{ic} / (R_{ic} + R_s)$$
(18)

and

$$v_o = A_D v_{sd} R_{ic} / (R_{ic} + R_s) + A_C v_{sd} R_{ic} \delta R / [2(R_{ic} + R_s)^2]$$
(19)

$$CMRR_{sys} = \frac{A_D/A_C + \delta R/[2(R_{ic} + R_s)]}{[A_D/A_C]\delta R/[2(R_{ic} + R_s)] + 1}$$

$$\approx \frac{CMRR_A}{CMRR_A\delta R/[2(R_{ic} + R_s)] + 1}$$
(20)

CMRRA is specified by the manufacturer

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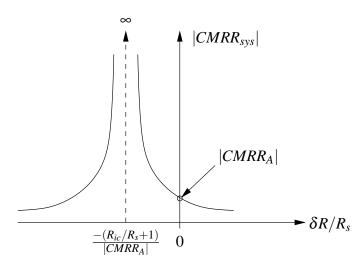


Figure 3: Differential amplifier CMRR magnitude vs fractional unbalance in source resistance.

#### **CMRR** - Special Cases

• If the thevenin source resistance are matched

$$CMRR_{sys} = CMRR_A \tag{21}$$

• When

 $\delta R/R_s = -2(R_{ic}/R_s+1)/CMRR_A$ 

then

$$CMRR_{sys} \rightarrow \infty$$

## Source Resistance Asymmetry

See Figure 3 We can increase the systems CMRR by adding an external resistance.

# 3 Operational Amplifiers

# Typical Op-Amp

#### **Open-loop Transfer Function**

$$A_D = \frac{v_o}{v_i - v'_i} = \frac{k_{ov}}{(\tau_1 s + 1)(\tau_2 s + 1)}$$
(22)

- $f_1 = 1/(2\pi\tau_1)$  and  $f_2 = 1/(2\pi\tau_2)$ .
- To ensure stability  $|A_D(jf_2)| \ll 1$ .
- $f_1$  occurs at a relatively low value.
- Small-signal gain  $\times$  bandwidth product

$$GBWP \approx k_{ov}/(2\pi\tau_1) \tag{23}$$

• The unity gain of 0dB frequency of the open loop,  $f_T$ , is approximately equal to the *GBWP*.

• Slew rate  $\eta$ .

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## Notes

- Two pieces of information define are required to define the op-amp's open-loop characteristics.
  - DC open-loop gain  $k_{ov}$ , and
  - 0dB frequency,  $f_T$ .
- Closed-loop gain is limited by the open-loop gain.
- The higher the closed-loop gain is the smaller the bandwidth.
- To overcome this problem, it is better to cascade identical amplification stages rather than having one op-amp with a high gain.

# 3.1 Broadband Amplifier

Non-inverting Amplifier

See Figure 4A.

$$v_o = (v_s - v_i')\frac{k_{vo}}{\tau s + 1} \tag{24}$$

$$v_i' = v_o \frac{R_1}{R_1 + R_F} = \beta v_o \tag{25}$$

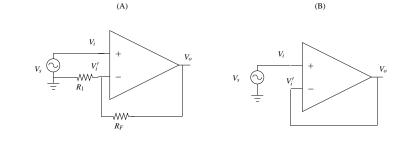


Figure 4: Non-inverting op-amp circuits

$$v_o/v_s = \frac{k_{ov}/(\tau s + 1)}{1 + \beta k_{ov}/(\tau s + 1)}$$
(26)

which, assuming that  $\beta k_{ov} \gg 1$  reduces to

$$v_o/v_s = \frac{1/\beta}{s(\tau/(\beta k_{ov})) + 1}$$
 (27)

$$GBWP = [k_{ov}/(1+\beta k_{ov})][(\beta k_{ov})/(2\pi\tau)] = k_{ov}/(2\pi\tau)$$
(28)

If  $R_1$  is infinite and  $R_F$  is shortcircuit, then we obtain a voltage follower that can be used as impedance isolation, see Figure 4B.

#### Inverting Amplifier and Summer

See Figure 5. The gain for the *k*th inverting input is

$$v_o/v_{sk} = \frac{-(G_1/G_F)}{s(\tau \sum G/(k_{ov}G_F) + 1)}$$
(29)

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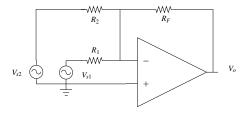


Figure 5: Summing op-amp