EE 521: Instrumentation and Measurements

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1. **Integrators and Differentiators**

2. **Active Filters**
   - Sallen-Key
   - Biquad Active Filters
   - Generalized Impedance Converter Active Filters

Integrators

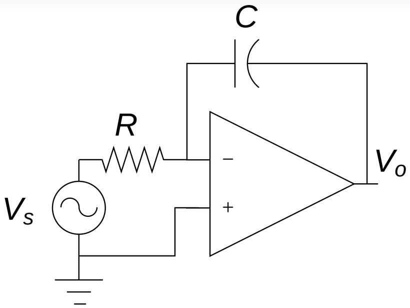

Figure: Integrator

Integrators

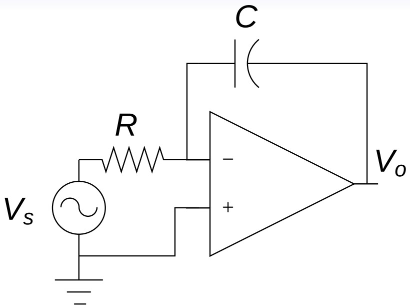

Figure: Integrator

- Drifts if there is dc or average levels in the signal.
- Will also drift due to current biases in the op-amp.

Differentiator

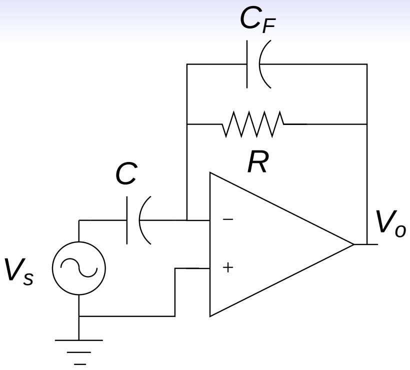

Figure: Practical differentiator

Differentiator

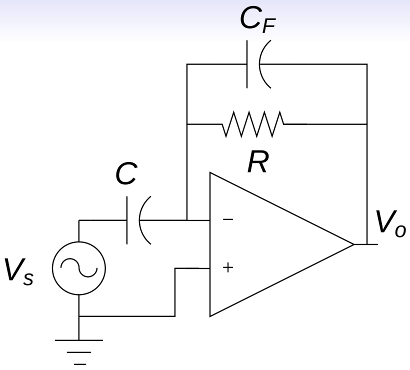

Figure: Practical differentiator

- dc or average levels in the signal and op-amp biases pose no issues.
- High frequency issues.

Common Filter Types

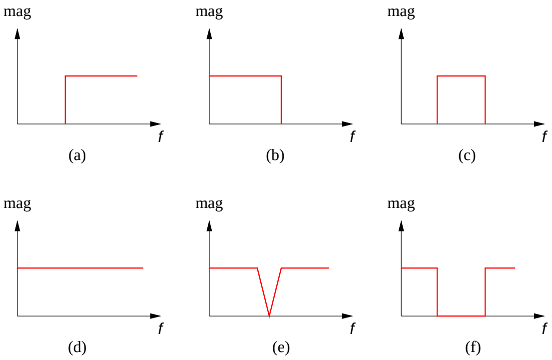

Figure: Sample of ideal filters: (a) lowpass; (b) highpass; (c) bandpass; (d) allpass; (e) notch; (f) bandstop

Active Filters - Generalized Circuit

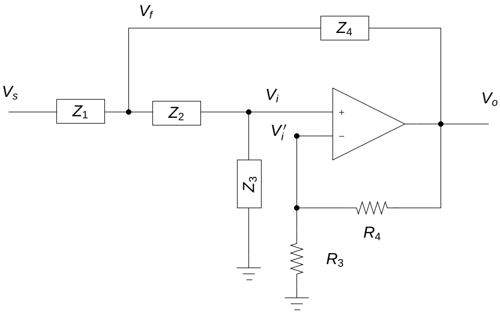

Figure: Generalized Sallen-Key Circuit

Non-Ideal Transfer Function

$$\frac{V_o}{V_s} = \left(\frac{c}{d}\right)\left[\frac{1}{1 + \frac{1}{A_D(f)b} - \frac{d}{b}}\right] \tag{1}$$

where

$$b = \frac{R_3}{R_3 + R_4}$$

$$c = \frac{Z_2Z_3Z_4}{Z_2Z_3Z_4 + Z_1Z_2Z_4 + Z_1Z_2Z_3 + Z_2Z_2Z_4 + Z_2Z_2Z_1}$$

$$d = \frac{Z_1Z_2Z_3}{Z_2Z_3Z_4 + Z_1Z_2Z_4 + Z_1Z_2Z_3 + Z_2Z_2Z_4 + Z_2Z_2Z_1}$$

Ideal Transfer Function

$$\frac{1}{A_D(f)b} \approx 0$$

$$\frac{V_o}{V_s} = \frac{K}{\frac{Z_1 Z_2}{Z_3 Z_4} + \frac{Z_1}{Z3} + \frac{Z_2}{Z_3} + \frac{Z_1(1-K)}{Z_4} + 1} \tag{2}$$

where

$$K = \frac{R_3 + R_4}{R_3} \tag{3}$$

Quadratic Form

$$s^2/\omega_n^2 + s_2\zeta/\omega_n + 1$$

where ω_n is the natural frequency and ζ is the damping factor. Also, $Q = 1/(2\zeta)$.

Low-Pass Filter

$$Z_1 = R_1, \quad Z_2 = R_2, \quad Z_3 = \frac{1}{sC_1}, \quad Z_4 = \frac{1}{sC_2}, \quad K = 1 + \frac{R_4}{R_3} \tag{4}$$

$$\frac{V_o}{V_s} = \frac{K}{s^2(R_1R_2C_1C_2) + s(R_1C_1 + R_2C_1 + R_1C_2(1-K)) + 1} \tag{5}$$

let

$$\omega_n^2 = \frac{1}{R_1R_2C_1C_2}, \quad \zeta^2 = \frac{R_1R_2C_1C_2}{(R_1C_1 + R_2C_1 + R_1C_2(1-K))^2}$$

High-Pass Filter

$$Z_1 = \frac{1}{sC_1}, \quad Z_2 = \frac{1}{sC_2}, \quad Z_3 = R_1, \quad Z_4 = R_2, \quad K = 1 + \frac{R_4}{R_3} \tag{6}$$

$$\frac{V_o}{V_s} = \frac{K(s^2(R_1R_2C_1C_2))}{s^2(R_1R_2C_1C_2) + s(R_2C_2 + R_2C_1 + R_1C_2(1-K)) + 1} \tag{7}$$

let

$$\omega_n^2 = \frac{1}{R_1R_2C_1C_2}, \quad \zeta^2 = \frac{R_1R_2C_1C_2}{(R_2C_2 + R_2C_1 + R_1C_2(1-K))^2}$$

Bandpass Filter

$$Z_1 = R_1, \quad Z_2 = \frac{1}{sC_1}, \quad Z_3 = R_2, \quad Z_4 = R_3||C_2, \quad K = 1 + \frac{R_4}{R_3} \tag{8}$$

Biquad Active Filter

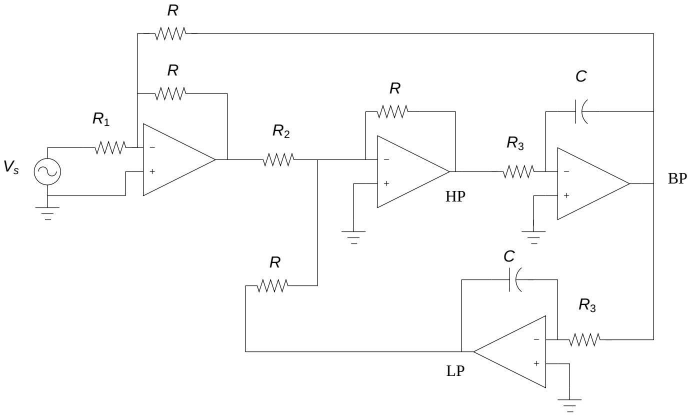

Figure: Biquad Active Filter

Notch filter

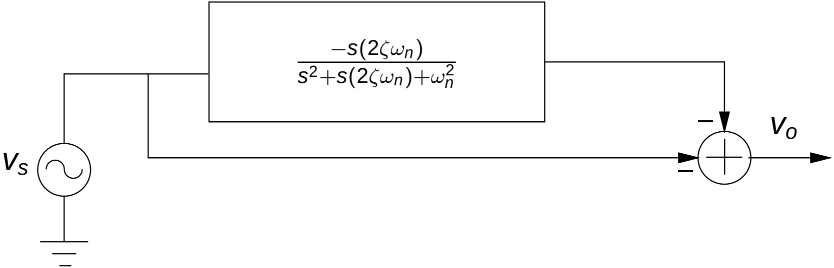

Figure: Biquad notch filter

Biquad notch filter

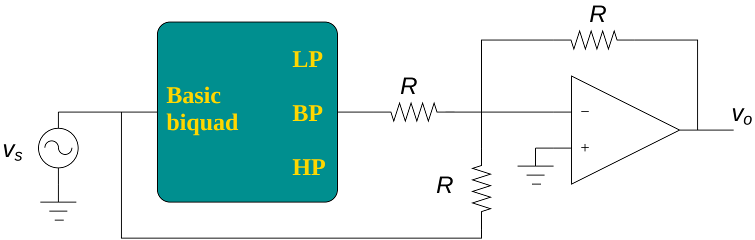

Figure: Biquad notch filter

where $R_1 = R$.

Biquad allpass filter

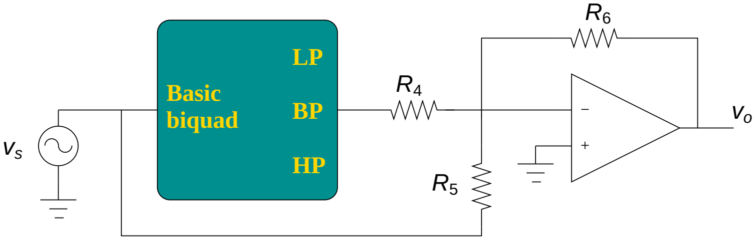

Figure: Biquad allpass filter

where $R_1 = R$, $R_5 = 2R_4$, and $R_6 = R_5$.

Generalized Impedance Converter (GIC)

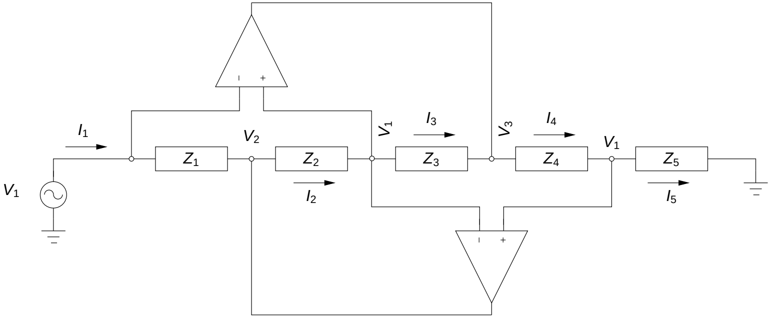

Figure: Generalized Sallen-Key Circuit

GIC driving point impedance

$$Z_{11}(s) = V_1/I_1 = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} \quad (9)$$

- If $Z_2 = 1/(sC_2)$ and the rest of the Zs are resistors

$$Z_{11} = s(C_2 R_1 R_3 / R_4) \quad (10)$$

where the equivalent inductance is given by

$$L_{eq} = C_2 R_1 R_3 / R_4 \quad (11)$$

- If Z_1 and Z_5 are made capacitors, then

$$Z_{11} = -1/(Ds^2) \quad (12)$$

where the D element is

$$D = C_1 C_5 R_2 R_4 / R_3 \quad (13)$$

Filter examples using GIC

- Bandpass filter

$$V_o/V_s = \frac{sL_{eq}/R}{s^2CL_{eq} + (sL_{eq}/R) + 1} \tag{14}$$

- Low-pass filter

$$V_o/V_s = \frac{1}{s^2RD + sRC + 1} \tag{15}$$

Bandpass Filter

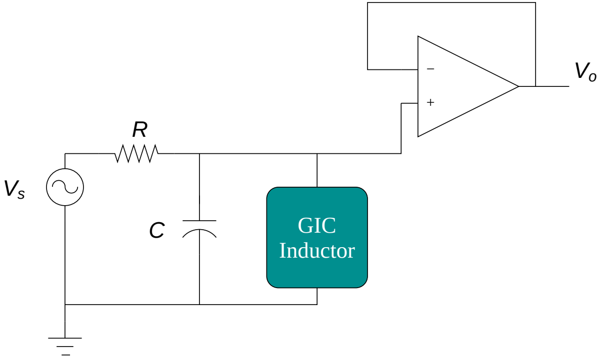

Figure: Bandpass filter using GIC

Low-pass Filter

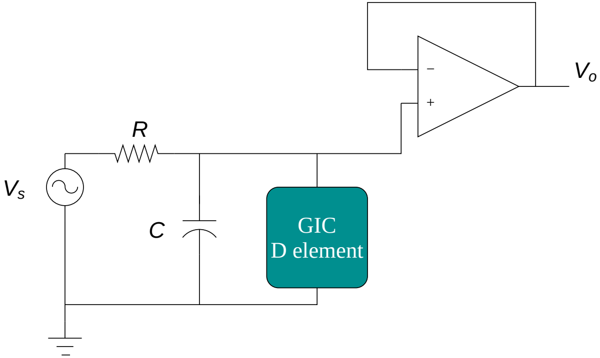

Figure: Lowpass filter using GIC