Lecture 4

Noise and Coherent Interference in Measurements

EE 521: Instrumentation and Measurements

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1 Physical Noise Sources

1.1 Thermal Noise

Thermal or Johnson Noise

Thermal or *Johnson* noise arises from the random motion of charge carriers in a conducting or semi-conducting medium. 4.3

Noisy Resistor

Nyquist's theorem

$$v_{rms}^2 = < v_n^2(t) > = 4kTRB \quad V^2 \tag{1}$$

represents the mean-squared noise voltage appearing across the terminals of a resistor of R ohms at temperature T Kelvin in the frequency band B hertz where $k = 1.38 \times 10^{-23}$J/K is Boltzmann's constant.

See Figure 1 4.4

Noisy Resistor

$$i_{rms}^2 = < i_n^2(t) > = \frac{< v_n^2(t) >}{R^2} = 4kTGB \quad A^2 \tag{2}$$

See Figure 2 4.5

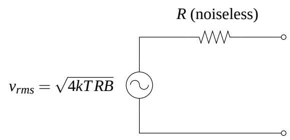

Figure 1: Thevenin equivalent circuit for a noisy resistor.

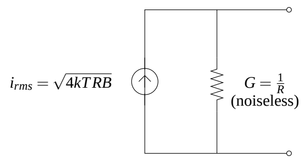

Figure 2: Norton equivalent circuit for a noisy resistor.

Nyquist's Formula

Noise computation can get considerably lengthy if the circuit contains many resistors. Using Nyquist's formula, the mean square noise voltage produced at the output terminal of any one-port network can be computed as

$$< v_n^2(t) >= 2kT \int_{-\infty}^{\infty} R(f)df \tag{3}$$

where $R(f)$ is the real part of the complex impedance seen looking back at into the terminals. If the networks contain only resistors

$$v_{rms}^2 = 4kTR_{eq}B \quad V^2 \tag{4}$$

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1.2 Shot and Flicker Noise

Shot Noise

Shot noise results from the discrete nature of current flow in electronic devices.

$$i_{rms}^2 =< i_n^2(t) >= 2eI_dB \quad A^2 \tag{5}$$

where $e = 1.6 \times 10^{-19}$C is the charge of and electron. 4.7

Flicker Noise or $1/f$ Noise

The power spectral density of flicker noise is characterized by $1/f$ dependency. 4.8

1.3 Available Power

See Figure 3

- Maximum power is transferred from a source with internal resistance R to a resistive load R_L if $R = R_L$, i.e., load is *matched* to the source.

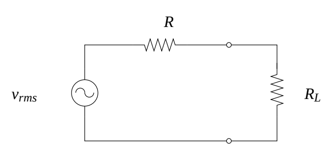

Figure 3: Thevenin equivalent for a source with load resistance R_L.

- The delivered power to the load, P_a, is known as *available power*.

$$P_a = \frac{1}{R}\left(\frac{1}{2}v_{rms}\right)^2 = \frac{v_{rms}^2}{4R} \tag{6}$$

- Assuming a noisy resistor, the available power is

$$P_{a,R} = \frac{4kTRB}{4R} = kTB \qquad W \tag{7}$$

2 System Noise Characterization

2.1 Noise Figure

Cascade of subsystems

See Figure 4

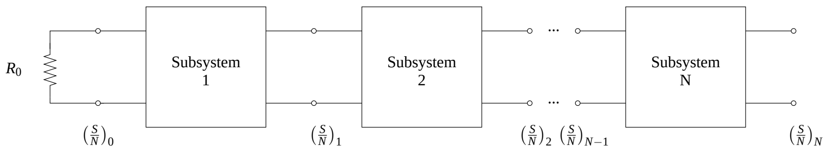

Figure 4: A system consisting of a cascade of subsystems.

See Figure 5

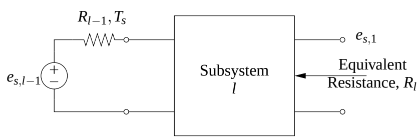

Figure 5: lth subsystems

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Noise Figure F

$$F = \frac{SNR_{in}}{SNR_{out}} \tag{8}$$

Looking at the lth subsystem

$$\left(\frac{S}{N}\right)_l = \frac{1}{F_l}\left(\frac{S}{N}\right)_{l-1} \tag{9}$$

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Noise Figure F

$$F_l = 1 + \frac{P_{int,l}}{G_a k T_s B} \tag{10}$$

Computing the input SNR

$$\left(\frac{S}{N}\right)_{l-1} = \frac{P_{sa,l-1}}{P_{sa,l}} = \frac{e^2_{s,l-1}/(4R_{l-1})}{kT_sB} = \frac{e^2_{s,l-1}}{4kT_sR_{l-1}B} \tag{11}$$

and assuming things are matched,

$$P_{sa,l} = \frac{e^2_{s,l}}{4R_l} = G_a P_{sa,l-1} \tag{12}$$

The output SNR

$$\left(\frac{S}{N}\right)_l = \frac{P_{sa,l}}{P_{na,l}} = \frac{1}{F_1}\frac{P_{sa,l-1}}{P_{na,l-1}} \tag{13}$$

Consequently, the noise figure of the lth subsystem is given by

$$F_l = \frac{P_{na,l}}{G_a P_{na,l-1}} = 1 + \frac{P_{int,l}}{G_a k T_s B} \tag{14}$$

where $P_{int,l}$ is the internally generated noise power of the lth subsystem.
To provide a common standard assume $T_s = T_o = 290\text{K}$

$$F_l = 1 + \frac{P_{int,l}}{G_a k T_0 B} \tag{15}$$

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2.2 Effective Noise Temperature

Define the *effective noise temperature* T_e as

$$T_e = \frac{P_{int,l}}{G_a k B} \tag{16}$$

Note that this term is has the dimension of temperature, hence the name.
Therefore, we can write the noise figure as

$$F_l = 1 + \frac{T_e}{T_0} \tag{17}$$

and

$$T_e = (F_l - 1) T_0 \tag{18}$$

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Friis's Formula

$$F = F_1 + \frac{F_2 - 1}{G_{a_1}} + \frac{F_3 - 1}{G_{a_1} G_{a_2}} + \dots \tag{19}$$

$$T_e = T_{e_1} + \frac{T_{e_2}}{G_{a_1}} + \frac{T_{e_3}}{G_{a_1} G_{a_2}} + \dots \tag{20}$$

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