EE 521: Instrumentation and Measurements

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1 Physical Noise Sources

- Thermal Noise
- Shot and Flicker Noise
- Available Power

2 System Noise Characterization

- Noise Figure
- Effective Noise Temperature

Thermal or Johnson Noise

Thermal or *Johnson* noise arises from the random motion of charge carriers in a conducting or semiconducting medium.

Noisy Resistor

Nyquist's theorem

$$v_{rms}^2 = < v_n^2(t) > = 4kTRB \quad V^2 \tag{1}$$

represents the mean-squared noise voltage appearing across the terminals of a resistor of R ohms at temperature T Kelvin in the frequency band B hertz where $k = 1.38 \times 10^{-23}$J/K is Boltzmann's constant.

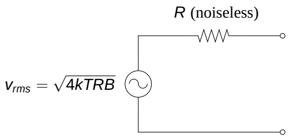

Figure: Thevenin equivalent circuit for a noisy resistor.

Noisy Resistor

$$i_{rms}^2 =< i_n^2(t) >= \frac{< v_n^2(t) >}{R^2} = 4kTGB \quad A^2 \tag{2}$$

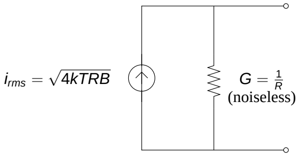

Figure: Norton equivalent circuit for a noisy resistor.

Nyquist's Formula

Noise computation can get considerably lengthy if the circuit contains many resistors. Using Nyquist's formula, the mean square noise voltage produced at the output terminal of any one-port network can be computed as

$$< v_n^2(t) >= 2kT \int_{-\infty}^{\infty} R(f) df \tag{3}$$

where $R(f)$ is the real part of the complex impedance seen looking back at into the terminals. If the networks contain only resistors

$$v_{rms}^2 = 4kTR_{eq}B \quad V^2 \tag{4}$$

Shot Noise

Shot noise results from the discrete nature of current flow in electronic devices.

$$i_{rms}^2 = < i_n^2(t) > = 2eI_dB \quad A^2 \tag{5}$$

where $e = 1.6 \times 10^{-19}$C is the charge of and electron.

Flicker Noise or $1/f$ Noise

The power spectral density of flicker noise is characterized by $1/f$ dependency.

- Maximum power is transferred from a source with internal resistance R to a resistive load R_L if $R = R_L$, i.e., load is *matched* to the source.

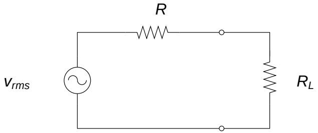

Figure: Thevenin equivalent for a source with load resistance R_L.

- The delivered power to the load, P_a, is known as *available power*.

$$P_a = \frac{1}{R}\left(\frac{1}{2}v_{rms}\right)^2 = \frac{v_{rms}^2}{4R} \quad (6)$$

- Assuming a noisy resistor, the available power is

$$P_{a,R} = \frac{4kTRB}{4R} = kTB \qquad W \quad (7)$$

Cascade of subsystems

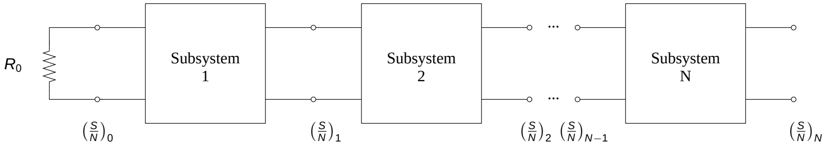

Figure: A system consisting of a cascade of subsystems.

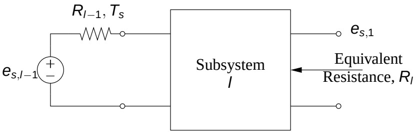

Figure: lth subsystems

Noise Figure F

$$F = \frac{SNR_{in}}{SNR_{out}} \tag{8}$$

Looking at the lth subsystem

$$\left(\frac{S}{N}\right)_l = \frac{1}{F_l}\left(\frac{S}{N}\right)_{l-1} \tag{9}$$

Noise Figure F

$$F_l = 1 + \frac{P_{int,l}}{G_a k T_s B} \tag{10}$$

where G_a is the available power gain of the lth subsystem, and $P_{int,l}$ is the internally generated noise power of the lth subsystem.

To provide a common standard assume $T_s = T_o = 290\text{K}$

$$F_l = 1 + \frac{P_{int,l}}{G_a k T_0 B} \tag{11}$$

Define the *effective noise temperature* T_e as

$$T_e = \frac{P_{int,l}}{G_a k B} \qquad (12)$$

Therefore, we can write the noise figure as

$$F_l = 1 + \frac{T_e}{T_0} \qquad (13)$$

and

$$T_e = (F_l - 1)T_0 \qquad (14)$$

Friis's Formula

$$F = F_1 + \frac{F_2 - 1}{G_{a_1}} + \frac{F_3 - 1}{G_{a_1} G_{a_2}} + \dots \quad (15)$$

$$T_e = T_{e_1} + \frac{T_{e_2}}{G_{a_1}} + \frac{T_{e_3}}{G_{a_1} G_{a_2}} + \dots \quad (16)$$