EE 491: Homework 1

Problem 1:

Sketch the single-sided and double-sided spectra of

1. $x(t) = 2\sin(2\pi t - \frac{1}{6}\pi)$

2. $y(t) = 2\sin(10\pi t - \frac{1}{6}\pi) + \cos(2\pi t)$

Problem 2:

Recall that

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}. \tag{1}$$

Show that

$$X_n = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} x(t) e^{-jn\omega_0 t} dt. \tag{2}$$

Hint: multiply both sides by $e^{-jm\omega_0 t}$ and integrate over $[t_0, t_0 + T_0]$, also remember that

$$\int_{t_0}^{t_0+T_0} e^{jn\omega_0 t} e^{-jm\omega_0 t} dt = \begin{cases} T_0 & ,n = m \\ 0 & ,n \neq m \end{cases} \tag{3}$$

Problem 3:

Determine the average power of $x(t) = 2\sin 100t$ using the following equations

1.
$$P = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt \tag{4}$$

   and

2.
$$P = \sum_{n=-\infty}^{\infty} |X_n|^2 \tag{5}$$

Problem 4:

Prove the following theorems:

1. Superposition : $a_1 x_1(t) + a_2 x_2(t) \leftrightarrow a_1 X_1(f) + a_2 X_2(f)$

2. Time-Delay : $x(t - t_0) \leftrightarrow X(f) e^{-j2\pi f t_0}$

3. Modulation : $x(t)cos(2\pi f_0 t) \leftrightarrow \frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$