# Lab 4
# IIR Filter Design by Pole-Zero Placement

### Dr. Aly El-Osery

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There are different ways to design IIR filters. In this lab we will address one of them, namely pole-zero placement. You will learn how to place the poles and zeros in order to achieve filter.

## 1 Introduction

The location of the poles and zeros of the filter determine the behavior of the filter. We will discuss pole-zero placement more formally in class. Looking at Figure 1, the magnitude of the frequency response at a specific value $\omega$ is equal to the product of the magnitude of the vectors of all the zeros to that point divided by the product of the magnitude of all the vectors of all the poles to that point. Therefore, as $\omega$ revolves around the unit circle, the magnitude of $H(\omega)$ increases when getting close to a pole, and decreases every time it gets closer to a zero. Poles and zeros close to each other cancel each others effect.

Figure 2, shows two examples of lowpass filters. They both perform as lowpass filters, but the have different performance. The one on the right will have sharper cutoff as well as having a magnitude of zero at $\omega = \pi$. On the other hand if we move the poles to the be around $\omega = \pi$ and move the zeros to around $\omega = 0$, we will have a highpass filter.

Now let's assume that we have the following filter:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

There are various ways to implement the above filter. One implementation is as follows

$$y[n] = x[n] + b_1 x[n-1] + b_2 x[n-2] - a_1 y[n-1] - a_2 y[n-2]$$

Another method is to introduce $W(z)$ and define it as

$$\frac{W(z)}{X(z)} = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

and

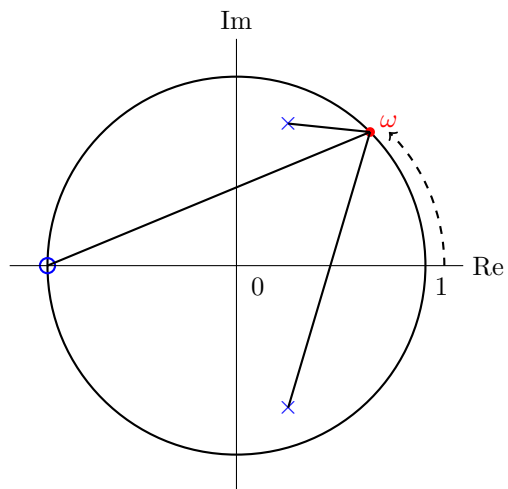$$\frac{Y(z)}{W(z)} = 1 + b_1 z^{-1} + b_2 z^{-2}$$

Figure 1: Determination of the frequency response from pole-zero location.
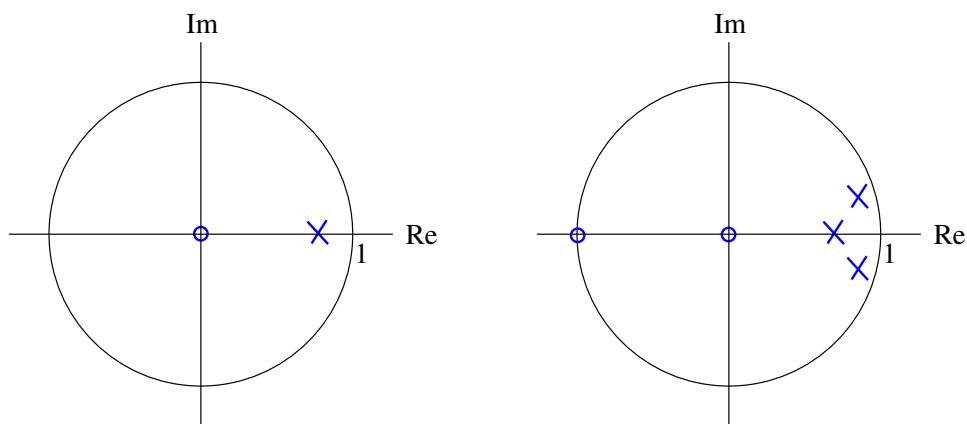


Figure 2: Two examples of lowpass filters.

Now, $y[n]$ may be expressed in terms of $w[n]$ as follows

$$y[n] = w[n] + b_1 w[n-1] + b_2 w[n-2]$$

where

$$w[n] = x[n] - a_1 w[n-1] - a_2 w[n-2]$$

This have the advantage of requiring less memory and is known as Direct form II.

# 2   Lab

Given a sampling rate of 48kHz. It is desired to design a second-order bandpass filter with center frequency, $f_0$, equal to 12kHz. Make sure that the filter has a unit gain at $f_0 = 12$kHz and zero at $f_0 = 24$kHz.

1. Where would you locate the poles and zeros?

2. Draw the pole-zero pattern of the above filter.

3. Use Matlab to determine the magnitude and the phase response of the filter you have designed.

4. Vary the location of the poles by bringing them closer or further away from the unit circle. Use Matlab to see the effects of your changes.

5. Start a new project.

6. Write a program that will implement the filter that you have designed. Output the input signal to one channel and the filtered signal through the other channel.

7. Connect a function generator to the board and vary the frequency. Record the magnitude and phase response of the filter.