Problem 1.1

(a) $x(t) = A \cos 2\pi f_o t \qquad -\infty < t < \infty$

power signal

$$P_x = \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} x^2(t)\,dt = \frac{A^2}{2}$$

(b) $x(t) = \begin{cases} A \cos 2\pi f_o t & -T_o/2 \leq t \leq T_o/2 \\ 0 & \text{elsewhere} \end{cases} \qquad T_o = 1/f_o$

energy signal

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-T_o/2}^{T_o/2} A^2 \cos^2 2\pi f_o t\,dt = \frac{A^2 T_o}{2}$$

(c) $x(t) = \begin{cases} A e^{-at} & t > 0 \quad a > 0 \\ 0 & \text{elsewere.} \end{cases}$

energy signal

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{0}^{\infty} A^2 e^{-2at}\,dt = \frac{A^2}{2a}$$

(d) $x(t) = \cos t + 5 \cos 2t \qquad -\infty < t < \infty$

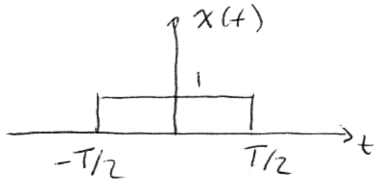
power signal $\qquad T_o = 2\pi$

$$P_x = \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} \cos^2 t + 10 \cos t \cos 5t + 25 \cos^2 2t\,dt$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{2} + \frac{25}{2}\,dt = 13$$

Problem 1.2

$$x(t) = \text{rect}(t/T)$$



$$\Psi(f) = |X(f)|^2 = T^2 \, \text{sinc}^2(fT)$$

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-T/2}^{T/2} dt = T$$

Problem 1.4

$$x(t) = 10\cos 10t + 20\cos 20t$$

$$2\pi f_0 = 10$$

$$\Rightarrow f_0 = 5/\pi$$

$$T_0 = \pi/5$$

$$P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x^2(t)\,dt$$

$$= \frac{5}{\pi}\int_{-\pi/10}^{\pi/10} 100\cos^2 10t + 400\cos^2 20t + 400\cos 10t\cos 20t\,dt$$

$$= 250\ \text{W}$$

Problem 1.6

$\mathcal{F}\{R(\tau)\} = G(f) =$ power spectral density which must be non-negative

(a) $x(\tau) = \begin{cases} 1 & -1 \leq \tau < 1 \\ 0 & \text{otherwise.} \end{cases}$

$x(\tau)$ is even
$x(0) \geq x(\tau)$
but $\mathcal{F}\{x(\tau)\}$ has negative values $\therefore$ not valid

(b) $x(\tau) = \delta(\tau) + \sin 2\pi f_0 \tau$

$x(\tau)$ not even $\therefore$ not valid.

(c) $x(\tau) = \exp(|\tau|)$

$x(\tau)$ is even
but $x(0) \not\geq x(\tau)$ $\therefore$ not valid.

(d) $x(\tau) = 1 - |\tau| \quad -1 \leq \tau \leq 1$, 0 elsewhere

$x(\tau)$ is even
$x(0) \geq x(\tau)$

$\mathcal{F}\{x(\tau)\} = 2 \operatorname{sinc}^2 f\tau > 0$

$\therefore$ valid

Problem 1.7.

(a) $X(f) = \delta(f) + \cos^2 2\pi f$

real, $P_X(f) \geq 0$, $P_X(-f) = P_X(f)$

$\therefore$ valid.

(b) $X(f) = 10 + \delta(f-10)$

real, $P_X(f) \geq 0$, $P_X(-f) \neq P_X(f)$

$\therefore$ not valid.

(c) $X(f) = e^{-2\pi|f-10|}$

real, $P_X(f) \geq 0$, $P_X(-f) \neq P_X(f)$

$\therefore$ not valid

(d) $X(f) = e^{-2\pi(f^2-10)}$

real, $P_X(f) \geq 0$, $P_X(-f) = P_X(f)$

$\therefore$ valid.

Problem 1.8

$$x(t) = A\cos(2\pi f_0 t + \Phi)$$

$$R_X(\tau) = \langle A\cos(2\pi f_0 t + \Phi)\, A\cos(2\pi f_0 (t+\tau) + \Phi)\rangle$$

$$R_X(\tau) = \frac{A^2}{2}\left[\cos 2\pi f_0 \tau + \langle\cos(4\pi f_0 t + 2\pi f_0 \tau + 2\Phi)\rangle\right]$$

$$= \frac{A^2}{2}\Big[\cos 2\pi f_0 \tau + \cos 2\pi f_0 \tau \overset{0}{\cancel{\langle \cos(4\pi f_0 t + 2\Phi)\rangle}}$$

$$+ \sin 2\pi f_0 \tau \overset{0}{\cancel{\langle \sin(4\pi f_0 t + 2\Phi)\rangle}}$$

$$= \frac{A^2}{2}\cos 2\pi f_0 \tau$$

$$P_X = R_X(0) = A^2/2$$

Problem 1.11

$$X(t) = A\cos(2\pi f_0 t + \phi)$$

$A,\ f_0$ const.

$\phi \sim \text{uniform}(0, 2\pi)$

$X(t)$ ... ergodic

(a) $$\langle X(t)\rangle = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A\cos(2\pi f_0 t + \phi)\,dt = 0$$

$$\langle X^2(t)\rangle = \frac{1}{T}\int_{-T_0/2}^{T_0/2} [A\cos(2\pi f_0 t + \phi)]^2\,dt$$

$$= \frac{A^2}{2}$$

(b) $$E\{X\} = \int_{-\infty}^{\infty} A\cos(2\pi f_0 t + \phi)\,p(\phi)\,d\phi$$

$$= \frac{1}{2\pi}\int_0^{2\pi} A\cos(2\pi f_0 t + \phi)\,d\phi = 0$$

$$E\{X^2\} = \frac{1}{2\pi}\int_0^{2\pi} [A\cos(2\pi f_0 t + \phi)]^2\,d\phi$$

$$= A^2/2$$

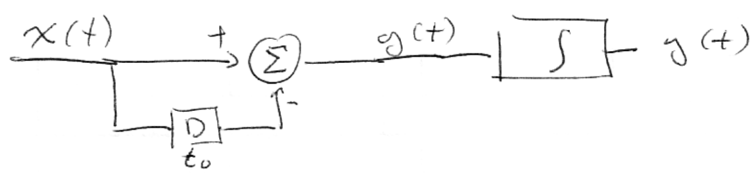
Problem 1.15

$$G_X(f) = 10^{-6} f^2$$

(a) $$P_X = 2 \int_0^{10\,kHz} 10^{-6} f^2 \, df = 667\ kW$$

(b) $$P_X = 2 \int_{5\,kHz}^{10\,kHz} 10^{-6} f^2 \, df = 583\ kW$$

Problem 1.19

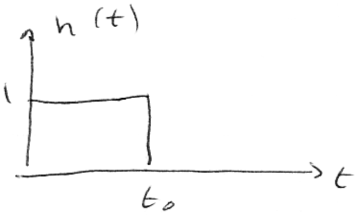


$$g(t) = x(t) - x(t - t_0)$$

impulse response $\Rightarrow x(t) = \delta(t)$

$$h(t) = \int_{-\infty}^{t} \delta(t) - \delta(t - t_0)\, d\tau$$

$$= u(t) - u(t - t_0)$$



Problem 1.20

$$G_x(f) = 10^{-4} \left\{ \frac{\sin(\pi(f-10^6)10^{-4})}{\pi(f-10^6)10^{-4}} \right\}^2$$

(a) half-power

$$\frac{1}{2} = \left(\frac{\sin(\pi f_0 10^{-4})}{\pi f_0 10^{-4}} \right)^2$$

$$\Rightarrow f_0 \approx 4.46k \text{ Hz}$$

$$BW = 2 f_0 \approx 9 \text{ kHz}$$

(b)
$$BW = 2 \int_0^\infty \left(\frac{\sin(\pi f 10^{-4})}{\pi f 10^{-4}} \right)^2 df$$

$$= \frac{2 \cdot 10^4}{\pi} \underbrace{\int_0^\infty \left(\frac{\sin x}{x} \right)^2 dx}_{\pi/2}$$

$$= 10 \text{ kHz}$$

(c) Null-to-Null

first null at $\pi f_0 10^{-4} = \pi$

$$\Rightarrow f_0 = 10 \text{ kHz}$$

$$BW = 2 f_0 = 20 \text{ kHz}$$

(d) 99% BW

$$0.99 = \frac{10^{-4} \int_0^{f_0} \left(\frac{\sin \pi f 10^{-4}}{\pi f 10^{-4}} \right)^2 df}{10^{-4} \int_0^{\infty} \left(\frac{\sin \pi f 10^{-4}}{\pi f \cdot 10^{-4}} \right)^2 df}$$

$$f_0 = 103 \text{ kHz} \Rightarrow BW = 2 f_0 = 206 \text{ kHz}$$

Problem 1.20 cont.

e) 35 dB BW

$35\,dB \Rightarrow 10^{-3.5} = 3.16 \times 10^{-4}$

peak occur ~~at~~ when $\pi f 10^{-4} = \frac{\pi}{2}(2k+1)$
when $\sin(\pi f 10^{-4})$ will be equal to unity.

$$10^{-3.5} \geq \frac{1}{(\pi/2\,(2k+1))^2}$$

$$\Rightarrow k = 18$$

$$\therefore \pi f_0 10^{-4} > \frac{\pi}{2}(2 \cdot 18 + 1) = 55.171$$

$$BW = 2 f_0 = 351.2 \text{ kHz}$$

(f) absolute $BW = \infty$