Problem 4.14

$P_B = 10^{-3}$ $\quad BW = 50\,kHz$ $\quad R_b = 100\,kbits/s$

raised-cosine $\quad r = 1$ $\quad$ no-ISI

$$W_{OSB} = (1+r)R_s = 2R_s$$

$$R_s = \frac{50\,kHz}{2} = 25\,k\,symbols/sec$$

$$k = \log_2 M = \frac{R}{R_s} = \frac{100\,kbits/s}{25\,ksymbols/s} = 4$$

$$\therefore M = 16$$

$$P_B \approx \frac{P_E}{\log_2 M} \Rightarrow P_E = (\log_2 M) P_B = 4 \times 10^{-3}$$

$$= 2Q\left[\sqrt{\frac{2E_s}{N_0}} \sin\frac{\pi}{M}\right] = 2Q(x)$$

$$Q(x) = 2 \times 10^{-3} \Rightarrow x = 2.88$$

(a) $$\Rightarrow \frac{E_s}{N_0} = 108.9 = 20.4\,dB$$

(b) $$\frac{E_b}{N_0} = \frac{E_s}{N_0} \cdot \frac{1}{k} = 27.2 = 14.3\,dB$$

Problem 4.16

coherent 8-ary FSK: $\frac{E_b}{N_0} = 8\,dB = 6.31$

$$P_E(M) = (M-1)\,Q\left(\sqrt{E_s/N_0}\right) = (M-1)\,Q\left(\sqrt{\frac{kE_b}{N_0}}\right)$$
$$\simeq 4.98 \times 10^{-5}$$

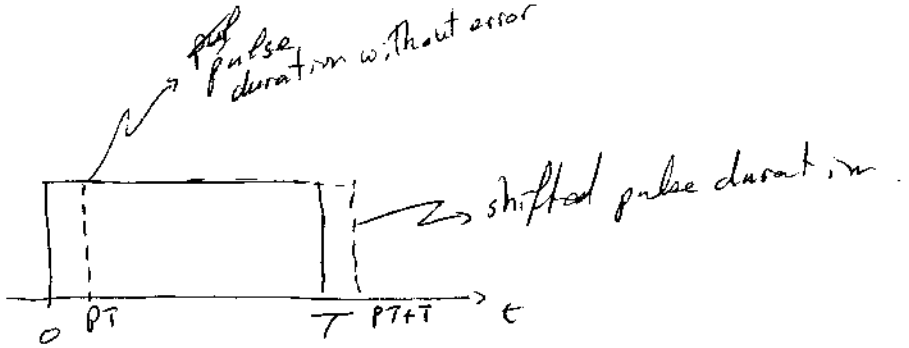
$$P_B = \frac{2^{k-1}}{2^k - 1} P_E = 2.85 \times 10^{-5}$$

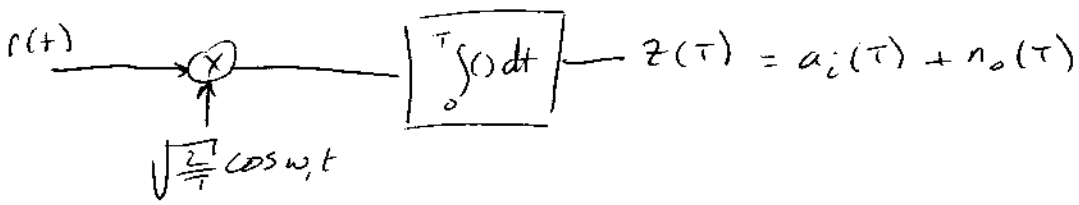
coherent 8-ary PSK: $\frac{E_b}{N_0} = 13\,dB \simeq 20$

$$P_E(M) = 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right) = 2Q\left(\sqrt{\frac{2kE_b}{N_0}}\sin\left(\frac{\pi}{M}\right)\right)$$
$$= 2Q(4.192) \simeq 2.9 \times 10^{-5}$$

$$P_B = \frac{P_E}{k} \simeq 9.7 \times 10^{-6}$$

Problem 4.17





$$s_1(t) = \sqrt{E}\sqrt{\frac{2}{T}}\cos\omega_1 t$$

$$s_2(t) = -\sqrt{E}\sqrt{\frac{2}{T}}\cos\omega_1 t$$

$$a_1(T) = \frac{2\sqrt{E}}{T}\left[\int_{\rho T}^{T}\cos^2\omega_1 t\,dt + \int_{T}^{T+\rho T} -\cos^2\omega_1 t\,dt\right]$$

assuming $s_1(t)$ is followed by $s_2(t)$ in which case, and because the integrator is still from 0 to T we get a portion of $s_2(t)$ along with $s_1(t)$

$$\Rightarrow a_1(T) = \sqrt{E}(1-2\rho)$$

similarly, if $s_2(t)$ is followed by $s_1(t)$

$$a_2(T) = -\sqrt{E}(1-2\rho)$$

if $s_1(t)$ is followed by $s_1(t)$ than the error does not matter and we get $\sqrt{E}$ or if $s_2(t)$ is followed by $s_2(t)$ we get $-\sqrt{E}$

now assuming that the probability of ~~the sent~~ $s_1(t)$ followed by $s_2(t)$ or $s_1(t)$ is equal.

$$\Rightarrow P_B = \frac{1}{2} Q\left(\sqrt{\frac{2E_b}{N_0}}\right) + \frac{1}{2} Q\left[\sqrt{\frac{2E_b}{N_0}}(1-2\rho)\right]$$

This is the error probability if $s_1(t)$ is followed by $s_1(t)$ or $s_2(t)$ is followed by $s_2(t)$

$s_1(t)$ followed by $s_2(t)$ or $s_2(t)$ followed by $s_1(t)$

(b) $\frac{E_b}{N_0} = 9.6\ dB = 9.12$

if $\rho = 0.2$

$\Rightarrow P_B = 2.6 \times 10^{-3}$

if $\rho = 0$

$\Rightarrow P_B = 10^{-5}$

(c) $P_B = 10^{-5} = \frac{1}{2} Q\left(\sqrt{\frac{2E_b}{N_0}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{2E_b}{N_0}}(1-2\rho)\right)$

$\Rightarrow \frac{E_b}{N_0} = 23.56 = 13.7\ dB$ (by trial & error)

need 4.1 dB more