Problem #6.1

$(n, k) = (8, 7)$ $\quad p = 10^{-2}$

$$P_{ud} = \sum_{j=1}^{n/2 \text{ (for } n \text{ even)}} \binom{n}{2j} p^{2j} (1-p)^{n-2j}$$

$$= \binom{8}{2} p^2 (1-p)^6 + \binom{8}{4} p^4 (1-p) + \binom{8}{6} p^6 (1-p)^2 + \binom{8}{8} p^8$$

$$= 2.6 \times 10^{-3}$$

Problem #6.2

$(n, k) = (24, 12)$ $\quad t = 2$ $\quad p = 10^{-3}$

$$P_M = \sum_{j=t+1}^{n} \binom{n}{j} p^j (1-p)^{n-j} = 1.98 \times 10^{-6}$$

Problem #6.3

$(n, k) = (127, 92)$ $\quad t = 3$

a) $P_M^u = 1 - (1 - 10^{-3})^{92} = 8.8 \times 10^{-2}$

b) $P_M^c = \sum_{j=4}^{127} \binom{127}{j} p^j (1-p)^{127-j} = 9.14 \times 10^{-6}$

Problem 6.4

$(n, k) = (24, 12) \qquad t = 2$

$$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{2 \times 10}\right) = 3.9 \times 10^{-6}$$

$$P_M^u = 1 - (1 - P_B)^{12} = 4.69 \times 10^{-5}$$

Code rate $= \frac{k}{n} = \frac{1}{2}$ $\Rightarrow$ twice the required R

$$\frac{E_c}{N_0} = \frac{1}{R}\frac{E_b}{N_0} = 5$$

$$P_c = Q\left(\sqrt{\frac{2E_c}{N_0}}\right) = Q\left(\sqrt{10}\right) = 7.8 \times 10^{-4}$$

$$P_M^c = \sum_{j=3}^{24} \binom{24}{j} p^j (1-p)^{24-j} \approx 1.02 \times 10^{-6}$$

Problem 6.5

$(n, k) = (24, 12)$ $\quad t = 2$

(a) $p_B^u = \frac{1}{2} e^{-1/2 \, E_b/N_0} = 1.76 \times 10^{-6}$

$p_M^u = 1 - (1 - p_B^u)^{12} \approx 2.11 \times 10^{-5}$

code rate $= 1/2 \Rightarrow$ R is doubled.

$\Rightarrow \frac{E_c}{N_0} = \frac{1}{2} \cdot 25.12 = 12.56$

$p_c = \frac{1}{2} e^{-1/2 \, E_c/N_0} \approx 9.2 \times 10^{-4}$

$p_M^c \approx \binom{24}{3} (9.2 \times 10^{-4})^3 (1 - 9.2 \times 10^{-4})^{21} \approx 1.56 \times 10^{-6}$

b) $E_b/N_0 = 10 \text{ dB} = 10$

$p_u = 3.36 \times 10^{-3}$

$p_M^u = 3.96 \times 10^{-2}$

code rate $= 1/2 \Rightarrow \frac{E_c}{N_0} = \frac{1}{2} E_b/N_0$

$p_c = 4.1 \times 10^{-2}$

$p_M^c \approx 5.7 \times 10^{-2}$ not much improvement.

Problem 6.8

$$(n, k) = (7, 4)$$

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

a)

msg	Codeword U=mG
0 0 0 0	0 0 0 0 0 0 0
0 0 0 1	1 1 0 0 0 0 1
0 0 1 0	0 1 1 0 0 1 0
0 0 1 1	1 0 1 0 0 1 1
0 1 0 0	1 0 1 0 1 0 0
0 1 0 1	0 1 1 0 1 0 1
0 1 1 0	1 1 0 0 1 1 0
0 1 1 1	0 0 0 0 1 1 1
1 0 0 0	1 1 1 0 0 0 0
1 0 0 1	0 0 1 1 0 0 1
1 0 1 0	1 0 0 1 0 1 0
1 0 1 1	0 1 0 1 0 1 1
1 1 0 0	0 1 0 1 1 0 0
1 1 0 1	1 0 0 1 1 0 1
1 1 1 0	0 0 1 1 1 1 0
1 1 1 1	1 1 1 1 1 1 1

b) $$H = \left[I_{n-k} \mid P^T \right] = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

c) $$S = r H = [1\,1\,0\,1\,1\,0\,1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} = [0\ 1\ 0]$$

There is an error received.

d) $$d_{min} = 3 \Rightarrow t = \left\lfloor \frac{d_{min} - 1}{2} \right\rfloor = 1$$

e) $$m = d_{min} - 1 = 2$$

problem 6.13

$(n, k) = (3, 1)$

correct all single errors $\Rightarrow t = 1 = \frac{d_{min} - 1}{2}$

$\therefore d_{min} = 3$

msg	codeword	
0	0 0 0	> distance = 3
1	1 1 1	

standard array

0 0 0	1 1 1
0 0 1	1 1 0
0 1 0	1 0 1
1 0 0	0 1 1