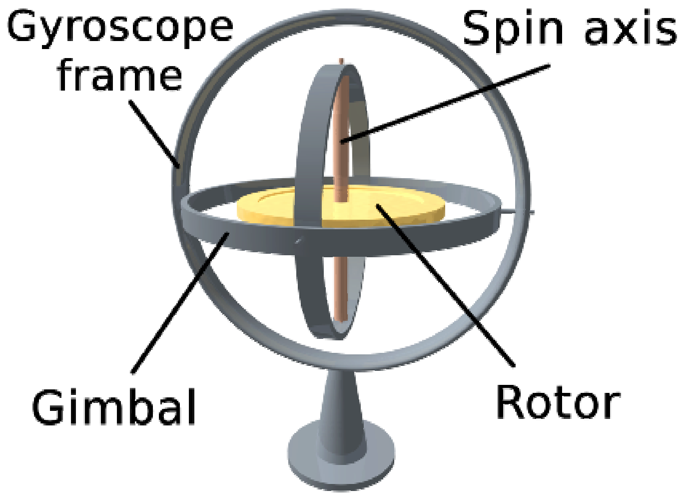
Inertial Sensors

Gyroscopes

- Gyroscope ⇔ Angular Rate Sensor
- Three main types
  - Spinning Mass
  - Optical
    - Ring Laser Gyros
    - Fiber Optic Gyros
  - Vibratory
    - Coriolis Effect devices
      - MEMS

Inertial Sensors

Gyroscopes – Spinning Mass

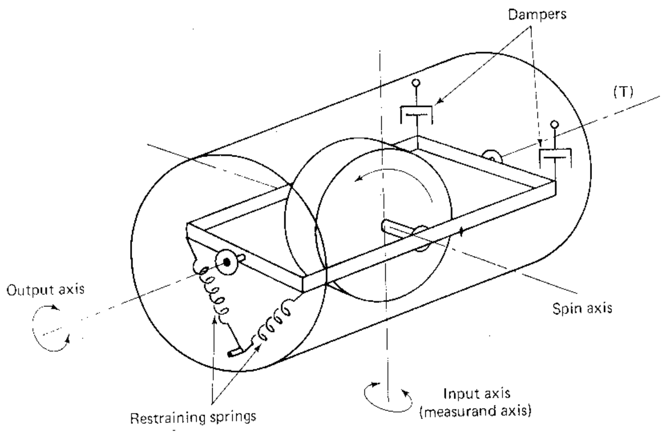


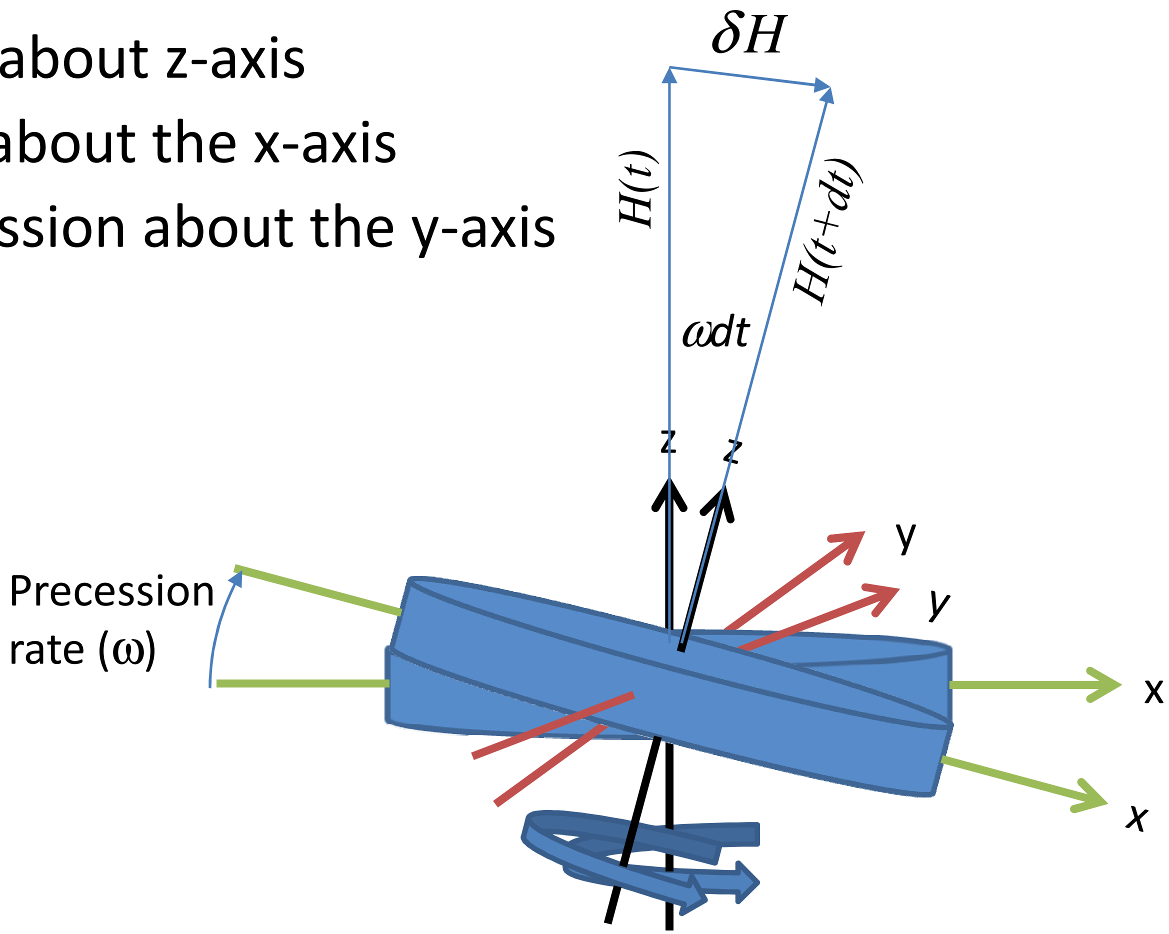
- Spinning Mass Gyroscopes
  - Conservation of Angular Momentum
  - The spinning mass will resist change in its angular momentum
  - Angular momentum
    - $H = I\omega$ (Inertia * Angular velocity)
  - By placing the gyro in a pair of frictionless gimbals it is free to maintain its inertial spin axis
  - By placing an index on the x-gimbal axes and y-gimbal axis two degrees of orientational motion can be measured

Inertial Sensors

Gyroscopes – Spinning Mass

- Precession
  - Disk is spinning about z-axis
  - Apply a torque about the x-axis
  - Results in precession about the y-axis
    - $\tau = \omega \times H$





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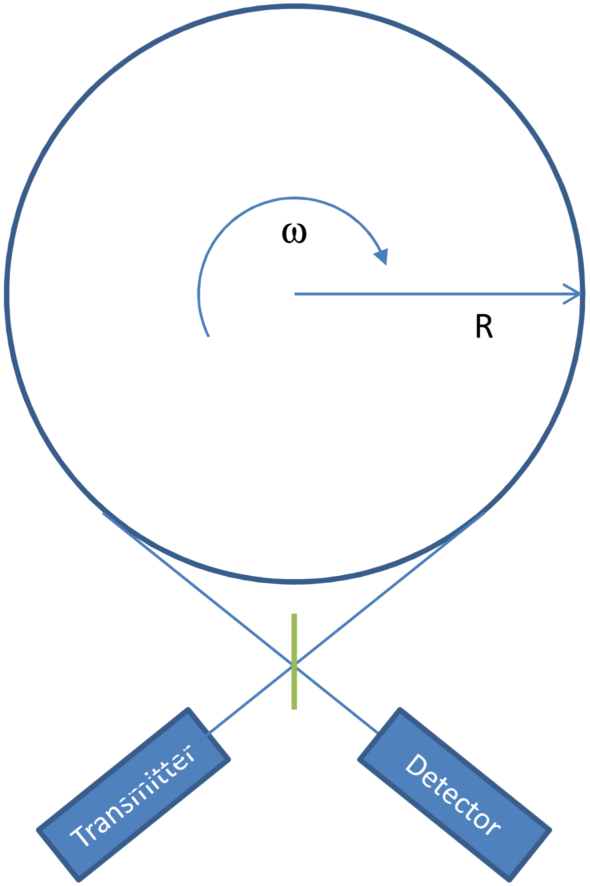
Gyroscopes - Optical

- Fiber Optical Gyro (FOG)
  - Basic idea is that light travels at a constant speed
  - If rotated (orthogonal to the plane) one path length becomes longer and the other shorter
  - This is known as the Sagnac effect
  - Measuring path length change (over a dt) allows ω to be measured



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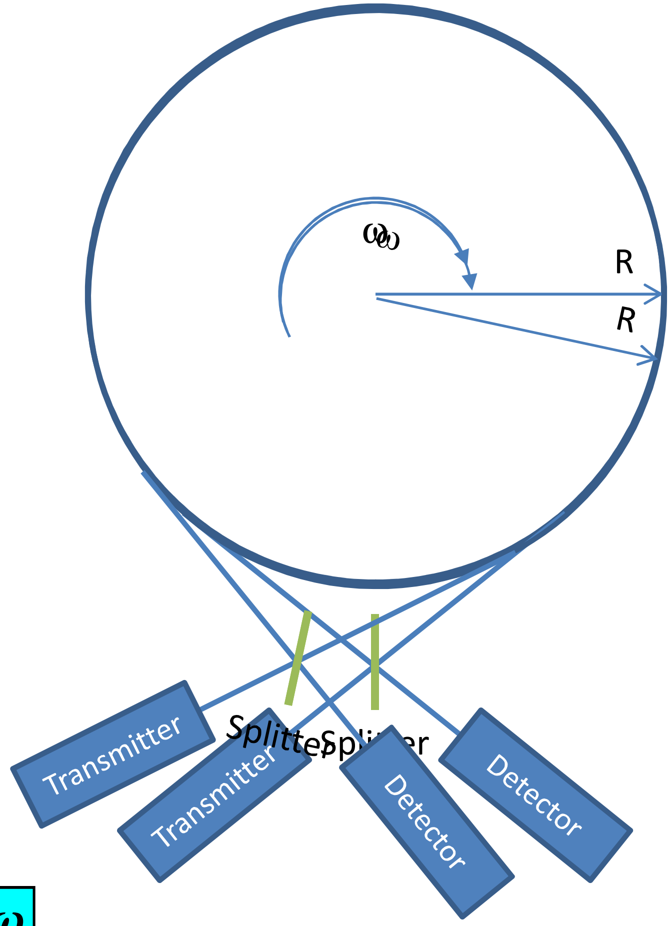
Gyroscopes - Optical

- Fiber Optical Gyro (FOG)
  - Measure the time difference betw the CW and CCW paths
  - CW transit time = t_{CW}
  - CCW transit time = t_{CCW}
  - $L_{CW} = 2\pi R + R\omega t_{CW} = ct_{CW}$
  - $L_{CCW} = 2\pi R - R\omega t_{CCW} = ct_{CCW}$
  - $t_{CW} = 2\pi R/(c - R\,\omega)$
  - $t_{CCW} = 2\pi R/(c + R\,\omega)$
  - With N turns
  - Phase

$$\Rightarrow \Delta t \simeq \frac{4\pi R^2\omega}{c^2}$$

$$\boxed{\Delta t \simeq \frac{N4A\omega}{c^2}}$$

$$\boxed{\phi_c \approx 2\pi\Delta t f_c = 2\pi\Delta t c / \lambda_0 = \frac{8\pi N A\omega}{c\lambda_0}}$$

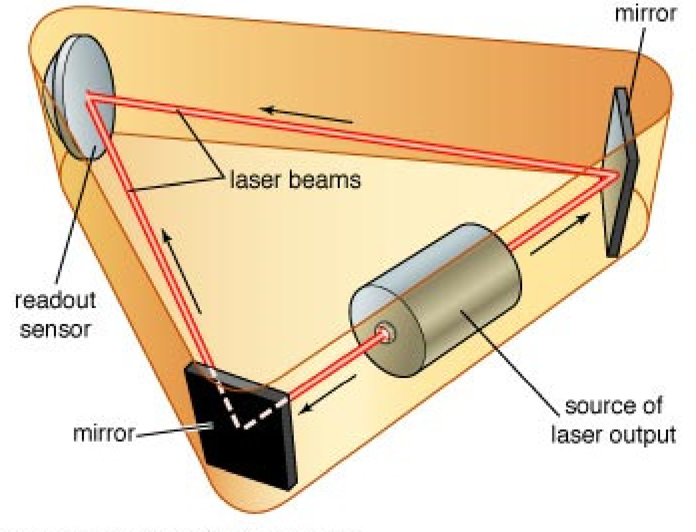


Inertial Sensors

Gyroscopes - Optical

- Ring Laser Gyro
  - A helium-neon laser produces two light beams, one traveling in the CW direction and the other in the CCW direction
  - When rotating
    - The wavelength in dir of rotation increases (decrease in freq)
    - The wavelength in opposite dir decreases (decrease in freq)
    - Similarly, it can be shown that

$$\Delta f \simeq \frac{4A\omega}{\lambda_0}$$



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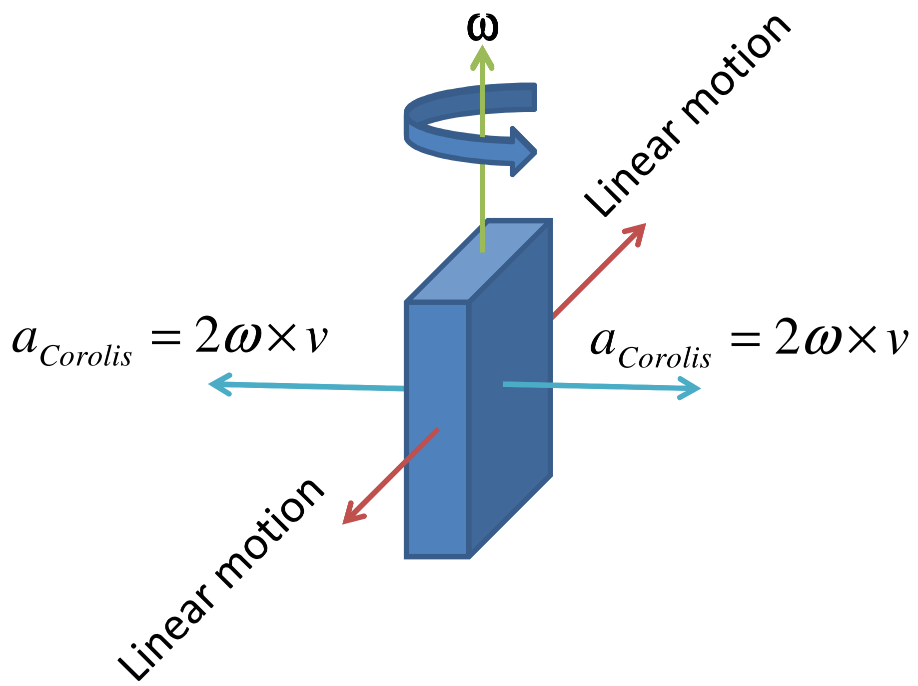
Gyroscopes - Vibratory

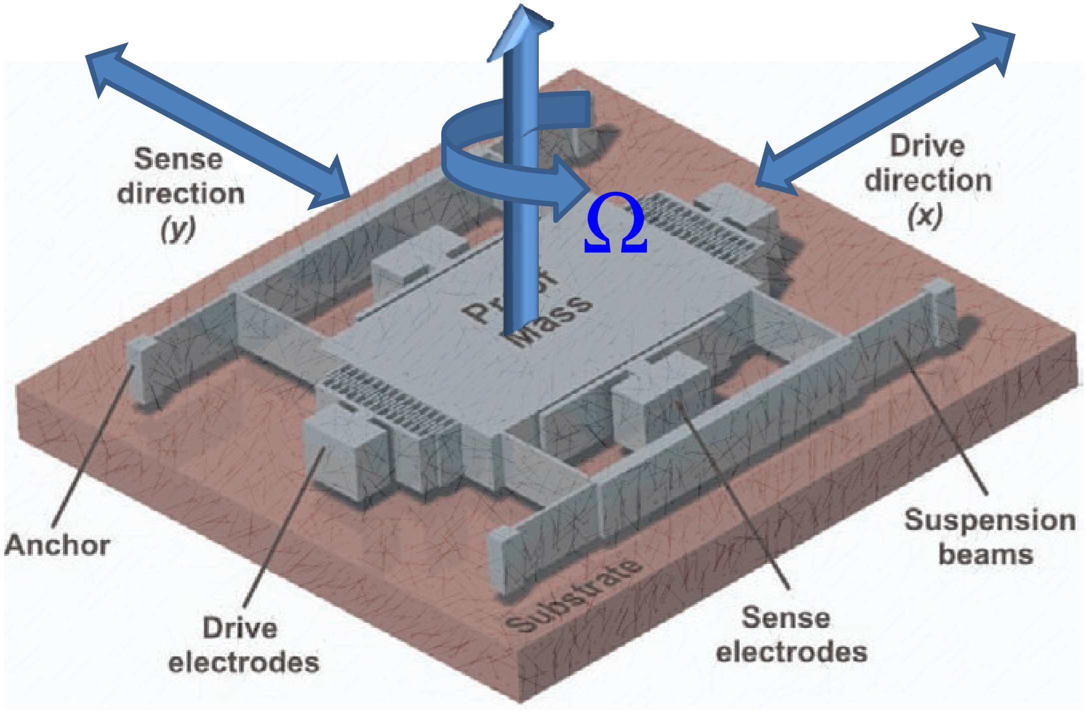
- Vibratory Coriolis Angular Rate Sensor
  - Virtually all MEMS gyros are based on this effect



Inertial Sensors

Gyroscopes - Vibratory

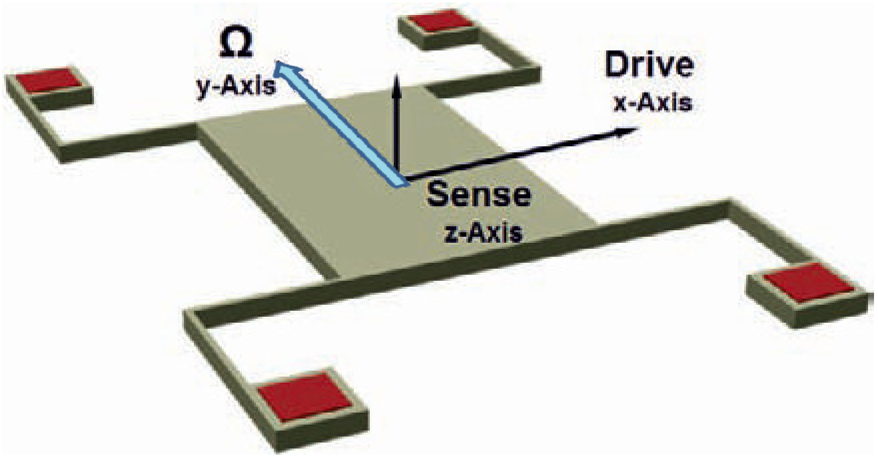
- Basic Planar Vibratory Gyro

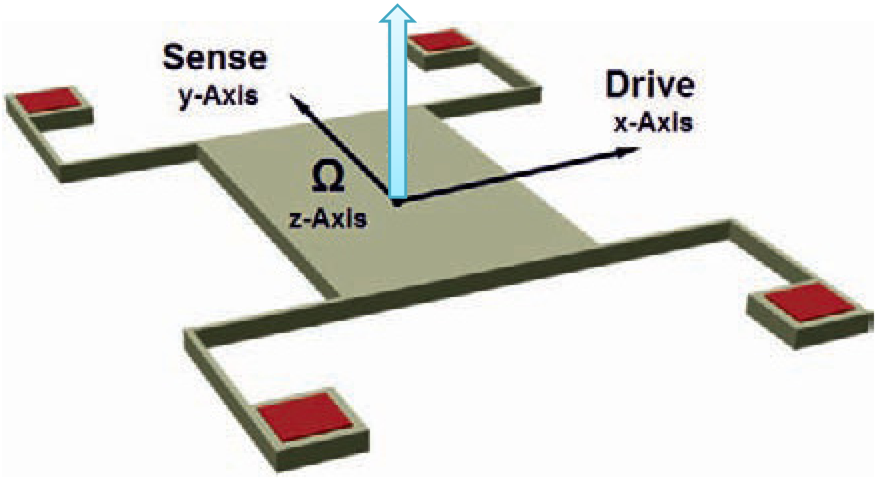


Inertial Sensors

Gyroscopes - Vibratory

- In plane sensing (left)
- Out of plane sensing (right)





www.ett.bme.hu/memsedu

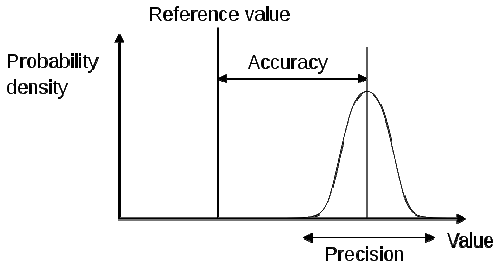
Inertial Sensors
Summary

- Accelerometers
    - Measure specific force of the body frame wrt the inertial frame in the body frame coordinates $\vec{f}_{ib}^{\,b}$
        - Need to subtract the acceleration due to gravity to obtain the motion induced quantity
    - In general, all points on a rigid body do <u>NOT</u> experience the same linear velocity
- Gyroscopes
    - Measure the inertial angular velocity
        - Essentially, the rate of change of orientation $\vec{\omega}_{ib}^{b}$
    - All points on a rigid body experience the same angular velocity

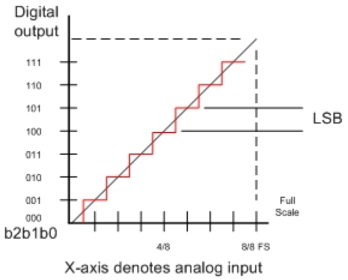
Inertial Sensor Modeling

Some Standard Terminology

- **Accuracy:**
  - Proximity of the measurement to the true value
- **Precision:**
  - The consistency with which a measurement can be obtained



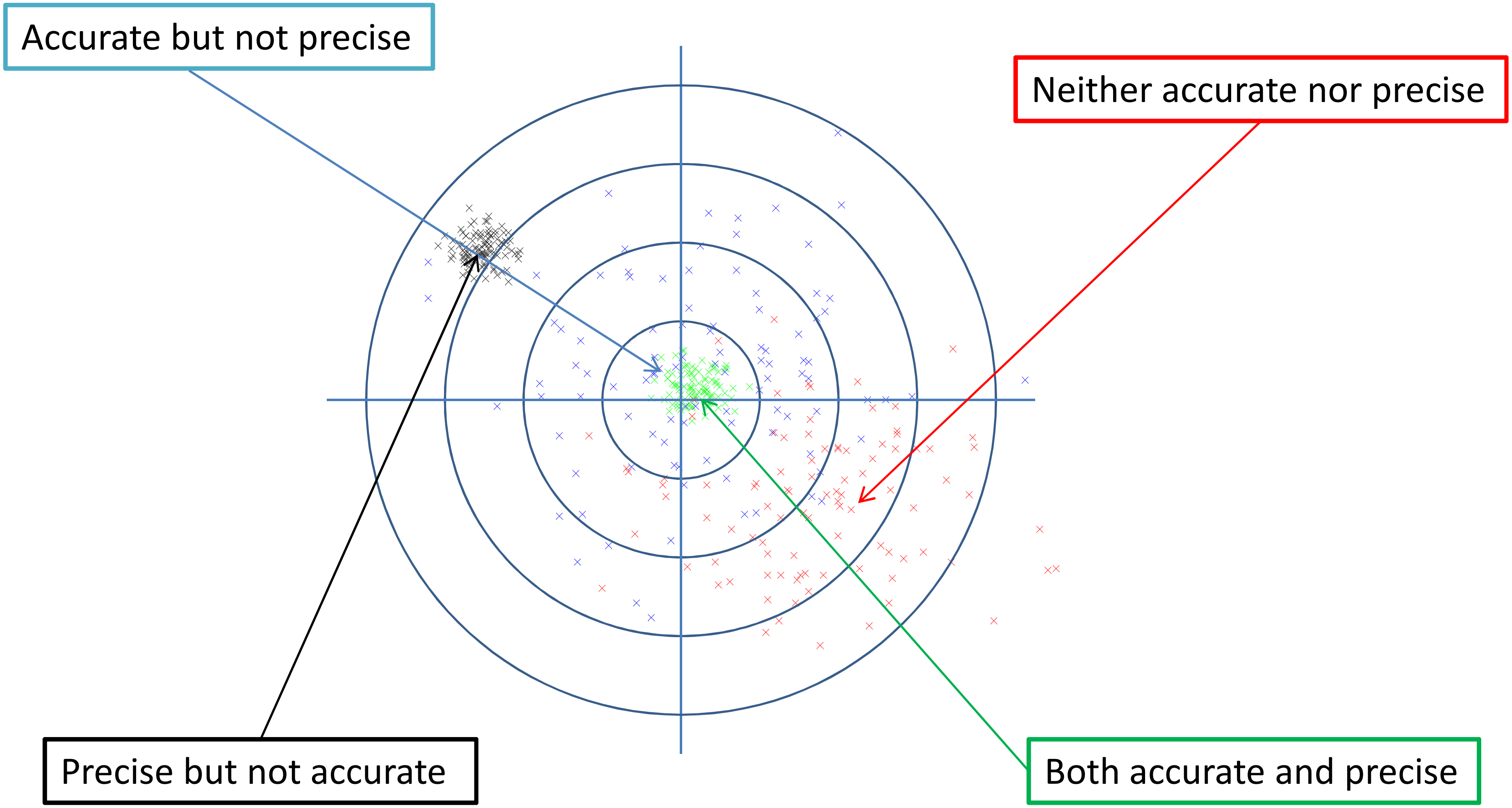
- **Resolution:**
  - The magnitude of the smallest detectable change.
- **Sensitivity:**
  - The ratio between the change in the output signal to a small change in input physical signal. Slope of the input-output fit line.



- **Linearity:**
  - The deviation of the output from a "best” straight line fit for a given range of the sensor

Inertial Sensor Modeling

Accuracy vs Precision



Accurate but not precise

Neither accurate nor precise

Precise but not accurate

Both accurate and precise

Inertial Sensors

Inertial Sensor Error Sources

- Bias – Often the most critical error source
  - Fixed Bias b_{FB}
    - Deterministic in nature and can be addressed by calibration
    - Often modeled as a function of temperature
  - Bias Stability b_{BS}
    - Varies from run-to-run as a random constant
  - Bias Instability b_{BI}
    - In-run bias drift – Typically modeled as a random walk

$$b_s = b_{FB} + b_{BS}$$

$$b_d = b_{BI}$$

$$\delta f = b_{a,BI} + b_{a,FB} + b_{a,BS} = b_a$$

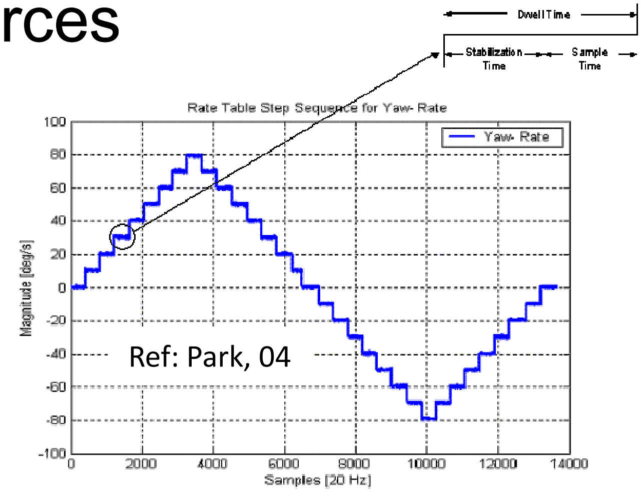
$$\delta\omega = b_{g,BI} + b_{g,FB} + b_{g,BS} = b_g$$

Gyro bias errors are a major INS error source

Inertial Sensors

Inertial Sensor Error Sources

- Scale Factor
  - Fixed Scale Factor Error
    - Deterministic in nature and can be addressed by calibration
    - Often modeled as a function of temperature
  - Scale Factor Stability
    - Varies from run-to-run as a random constant
    - Typically given in parts-per-million (ppm)

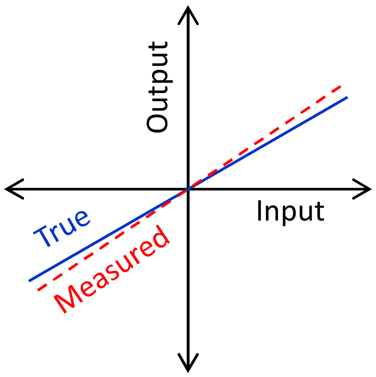


Ref: Park, 04

$$\delta f = s_a f$$

$$\delta\omega = s_a \omega$$

Scale Factor Error



The scale factor represents a linear approximation to the steady-state sensor response over a given input range – True sensor response may have some non-linear characteristics

Inertial Sensors

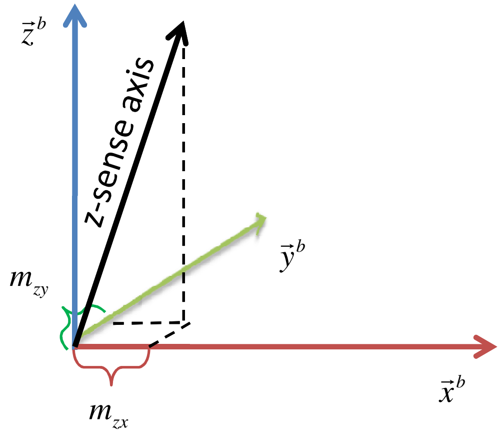
Inertial Sensor Error Sources

- Misalignment
  - Refers to the angular difference between the ideal sense axis alignment and true sense axis vector
    - A deterministic quantity typically given in milliradians

$$\delta f_z = m_{a,zx} f_x + m_{a,zy} f_y$$

$$\delta \omega_z = m_{g,zx} \omega_x + m_{g,zy} \omega_y$$

Normalized z-sense axis



  - Combining Misalignment & Scale Factor

$$\delta \vec{f} = \begin{bmatrix} s_{a,x} & m_{a,xy} & m_{a,xz} \\ m_{a,yx} & s_{a,y} & m_{a,yz} \\ m_{a,zx} & m_{a,zy} & s_{a,z} \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = M_a \vec{f}_{ib}^{\,b}$$

Inertial Sensors

Inertial Sensor Error Sources

- Cross-Axis Response
  - Refers to the sensor output which occurs when the device is presented with a stimulus which is vectorially orthogonal to the sense axis

Misalignment and cross-axis response are often difficult to distinguish – Particularly during testing and calibration activities

Inertial Sensors
Inertial Sensor Error Sources

- Other noise sources
  - Typically characterized as additive in nature
    - May have a compound form
      - White noise
        - Gyros: White noise in rate ⇒ Angle random walk
        - Accels: White noise in accel ⇒ Velocity random walk
      - Quantization noise
        - May be due to LSB resolution in ADC’s
      - Flicker noise
      - Colored noise

A more detailed discussion of noise will be given at a later date (25 March)

Inertial Sensors

Inertial Sensor Error Sources

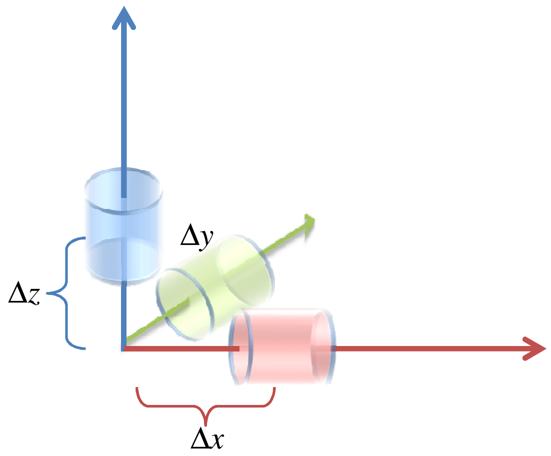
- Gyro Specific Errors
  - G-sensitivity
    - The gyro may be sensitive to acceleration
    - Primarily due to device mass assymetry
    - Mostly in Coriolis-based devices
  - G^2-Sensitivity
    - Anisoelastic effects
    - Due to products of orthogonal forces

$$\delta\vec{\omega}_{ib}^{b} = G_g \vec{f}_{ib}^{\,b}$$

Inertial Sensors
Inertial Sensor Error Sources

- Accelerometer Specific Errors
  - Axis Offset
    - The accel may be mounted at a lever-arm distance from the "center" of the Inertial Measurement Unit (IMU)
      - Leads to an "$\omega^2 r$" type effect

$$\delta f_x = \omega_y^2 \Delta x + \omega_z^2 \Delta x = \left(\omega_y^2 + \omega_z^2\right)\Delta x$$



Inertial Sensors

Modeling Inertial Sensors

- Accelerometer model

$$\tilde{f}_{ib}^{\,b} = \vec{f}_{ib}^{\,b} + \delta \vec{f}_{ib}^{\,b} = \vec{b}_a + \left(I + M_a\right) \vec{f}_{ib}^{\,b} + \vec{w}_a$$

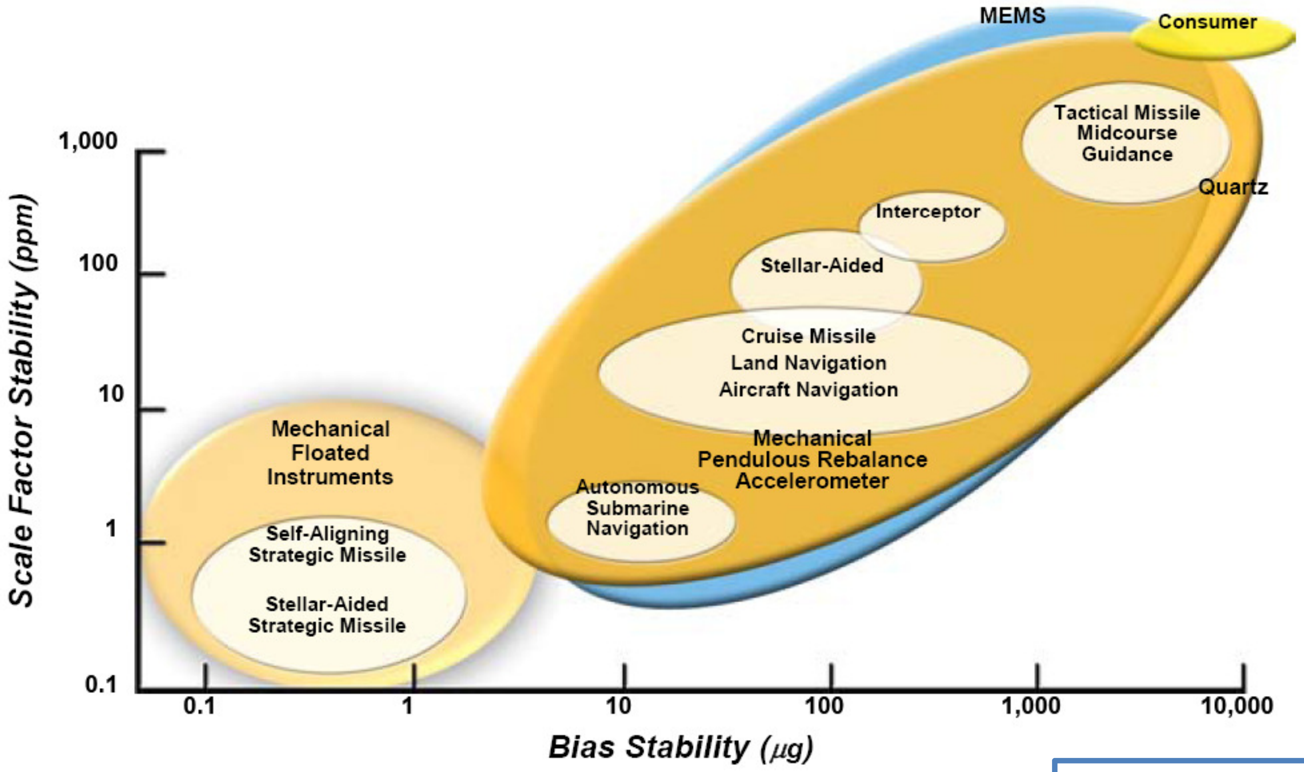
- Gyro Model

$$\tilde{\vec{\omega}}_{ib}^{b} = \vec{\omega}_{ib}^{b} + \delta\vec{\omega}_{ib}^{b} = \vec{b}_g + \left(I + M_g\right) \vec{f}_{ib}^{\,b} + G_g \vec{f}_{ib}^{\,b} + \vec{w}_g$$

- Typically, each measures along a single sense axis requiring three of each to measure the 3-tupple vector

Inertial Sensors

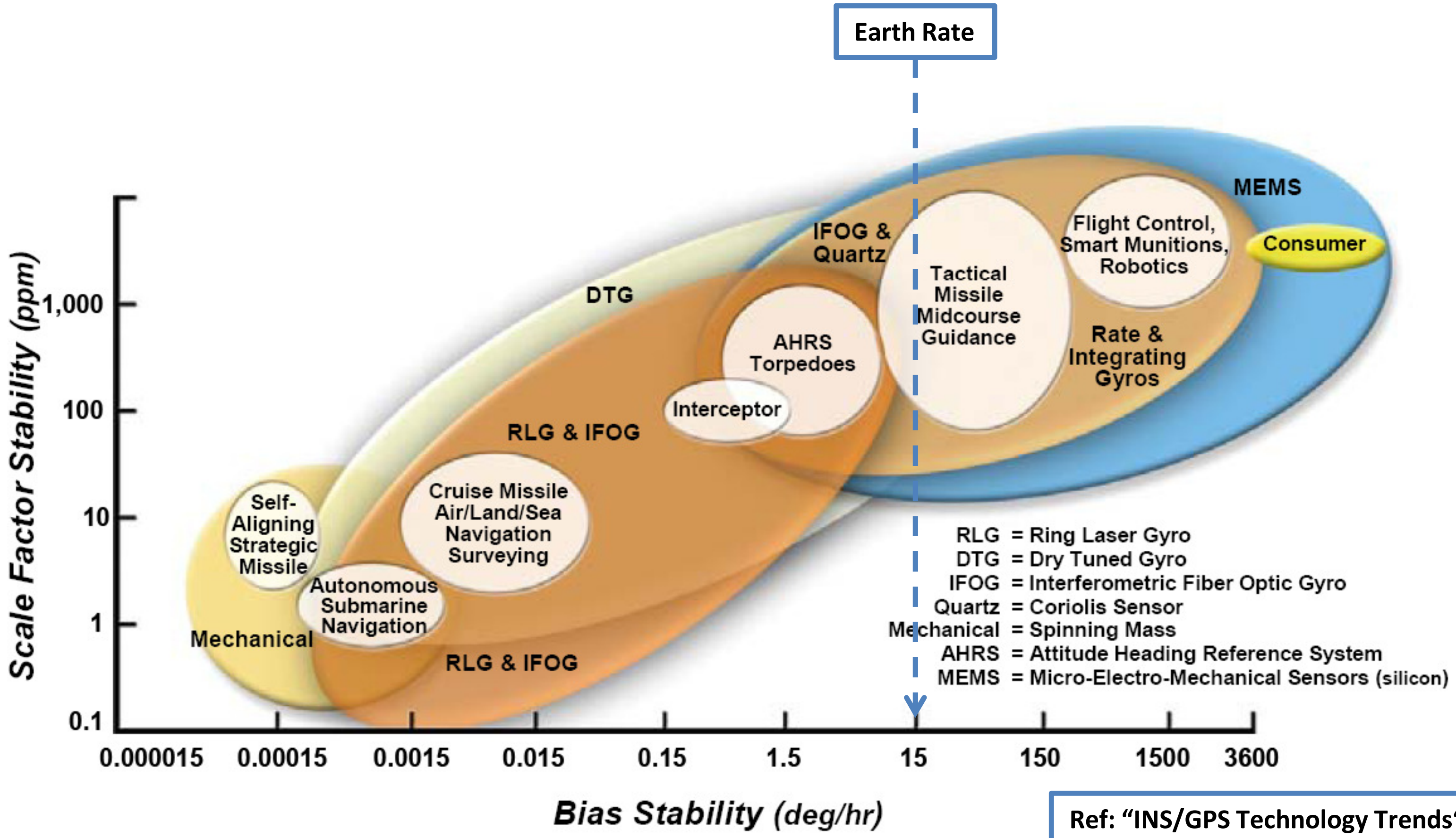
- Current Accelerometer Application Areas



Ref: "INS/GPS Technology Trends" by George T. Schmidt RTO-EN-SET-116(2010)

Inertial Sensors

- Current Gyro Application Areas

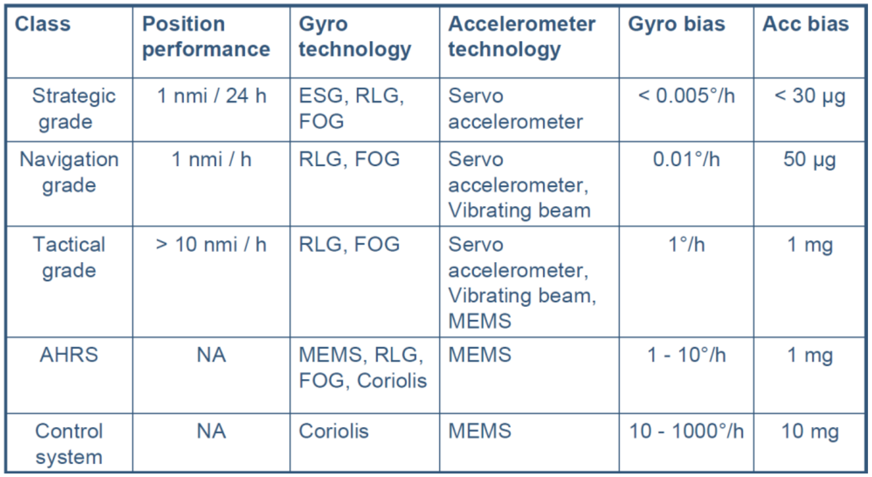


RLG = Ring Laser Gyro
DTG = Dry Tuned Gyro
IFOG = Interferometric Fiber Optic Gyro
Quartz = Coriolis Sensor
Mechanical = Spinning Mass
AHRS = Attitude Heading Reference System
MEMS = Micro-Electro-Mechanical Sensors (silicon)

Ref: "INS/GPS Technology Trends" by George T. Schmidt RTO-EN-SET-116(2010)

Inertial Sensors

Different “Grades” of Inertial Sensors



Class	Position performance	Gyro technology	Accelerometer technology	Gyro bias	Acc bias
Strategic grade	1 nmi / 24 h	ESG, RLG, FOG	Servo accelerometer	< 0.005°/h	< 30 µg
Navigation grade	1 nmi / h	RLG, FOG	Servo accelerometer, Vibrating beam	0.01°/h	50 µg
Tactical grade	> 10 nmi / h	RLG, FOG	Servo accelerometer, Vibrating beam, MEMS	1°/h	1 mg
AHRS	NA	MEMS, RLG, FOG, Coriolis	MEMS	1 - 10°/h	1 mg
Control system	NA	Coriolis	MEMS	10 - 1000°/h	10 mg

Ref: INS Tutorial, Norwegian Space Centre, 2008.06.09

Cost as a function of Performance and technology