The Global Positioning System (GPS)

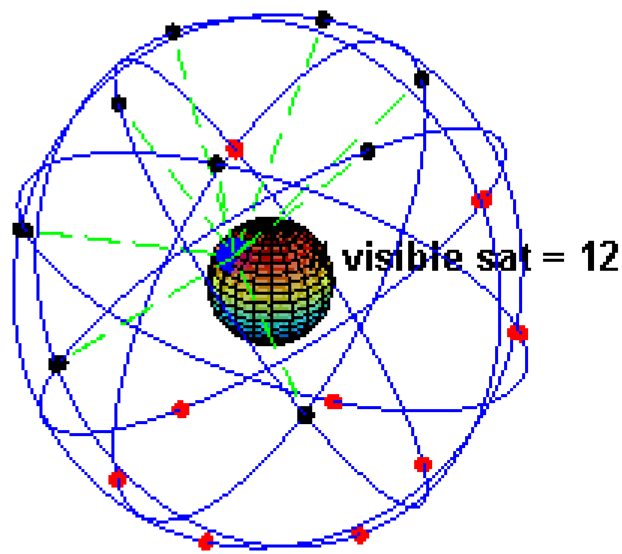
Friday 15 April 2011

The Global Positioning System
Overview

- The GPS is a Space-Based Global Navigation Satellite System (GNSS)
  - Space segment (satellites)
    - Satellites launched from 1989 (first) to 1994 (24th)
  - Control segment (ground station(s))
    - Master control segment, alternate, and monitors
  - User segment (receivers)
    - Both military and civilian
- GPS modernization is underway
  - First satellite transmission occurred in 2005
    - Additional civilian and military channels
    - Improved performance and reliability

The Global Positioning System
The Satellite Constellation

- Constellation of 24 satellites in 6 orbital planes
- Four satellites in each plane
- 20,200 km altitude at 55° inclination
  - Each satellite's orbital period is 12 hours
  - >6 satellites visible in each hemisphere



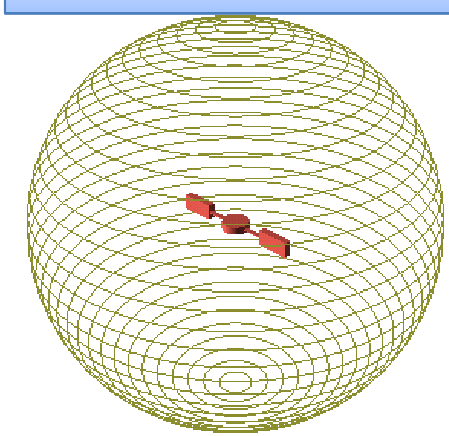
Courtesy of MATLAB™

Tutorial: www.trimble.com/gps/index.shtml

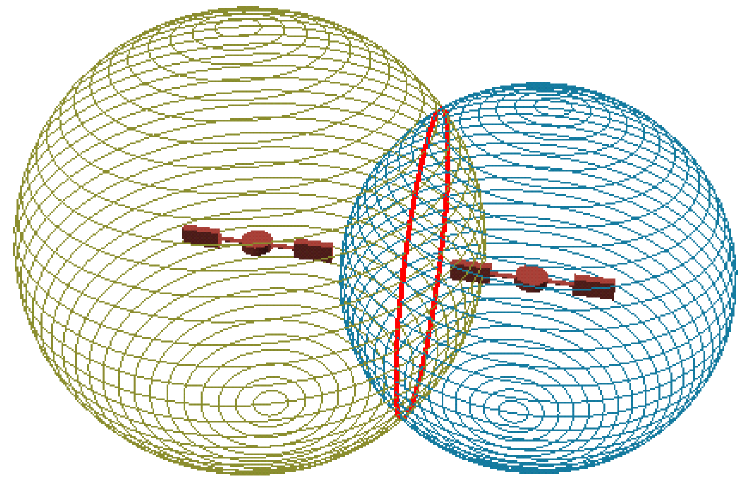
The Global Positioning System

Multilateration - Intersection of Spheres

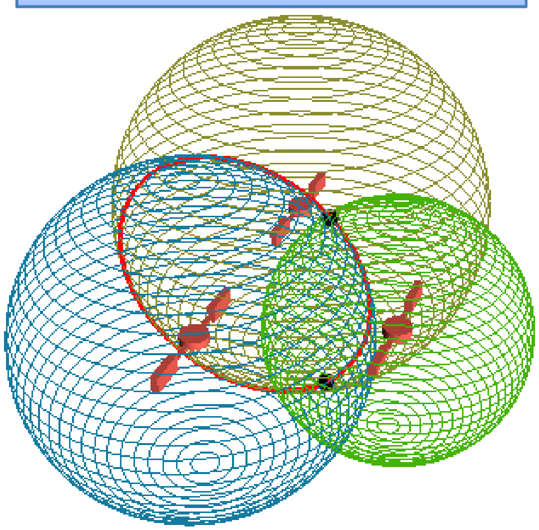
1 satellite – A sphere



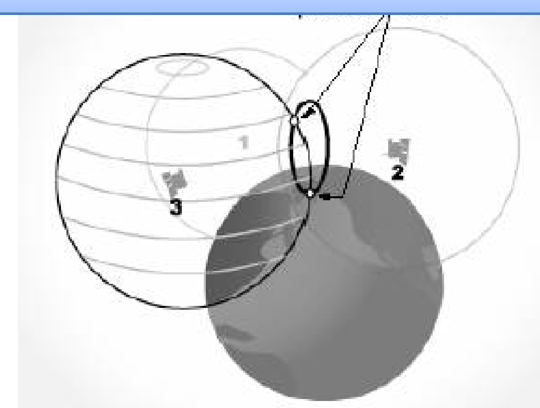
2 satellites – A circle



3 satellites – Two points



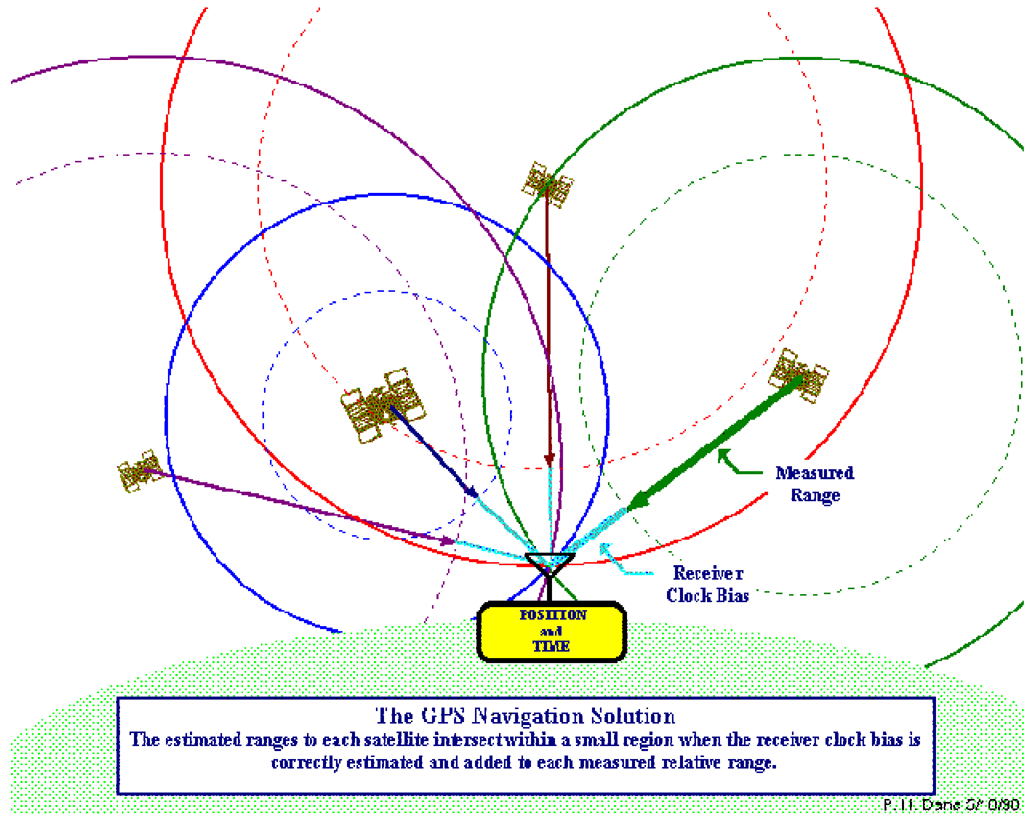
Only one point yields a “reasonable” soln



The Global Positioning System

Multilateration - Intersection of Spheres

- Basic Idea
  - Determine range to a given satellite via time-of-flight of an RF signal (*i.e.* speed of light)
  - Requires very precise time bases
    - Receiver clock bias



The Global Positioning System

Modulation Scheme

- Position is determined by the travel time of a signal from four or more satellites to the receiving antenna
  - Three satellites for X, Y, Z position, one satellite to solve for clock biases in the receiver

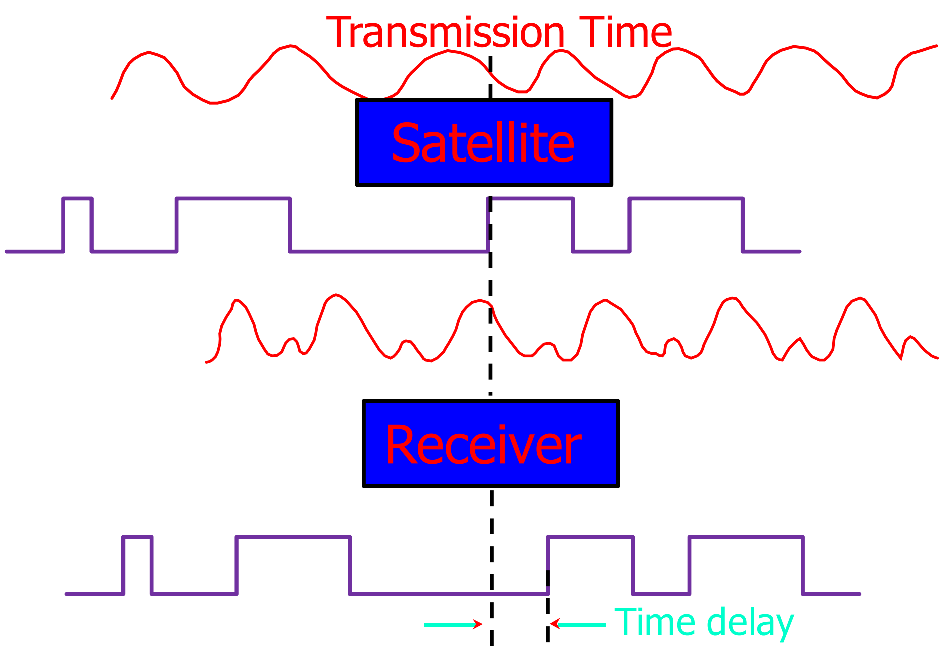
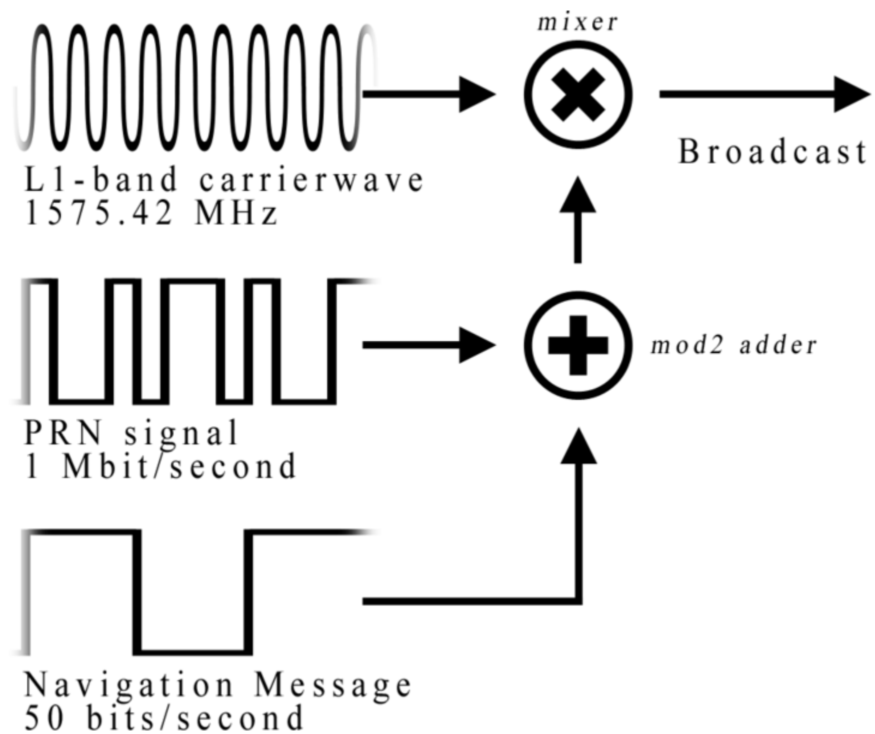


Image Source: NASA

The Global Positioning System

Modulation Scheme

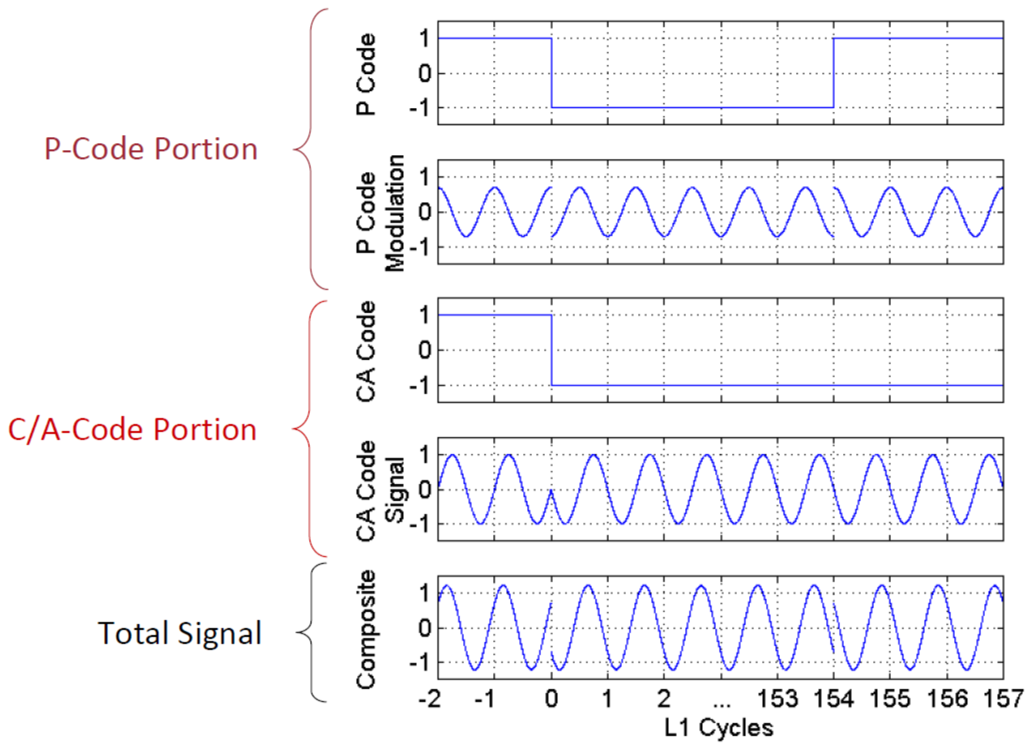
- The GPS employs quadrature Binary Phase Shift Keying (BPSK) modulation at two frequencies (CDMA)
  - L1 = 1,575.42 Mhz
    - $\lambda 1$ = 19 cm
  - L2 = 1,227.6 Mhz
    - $\lambda 1$ = 24 cm
- Two main PRN codes
  - C/A: Course acquisition
    - 10-bit 1 MHz
  - P: Precise
    - 40 bit 10 MHz
    - Encrypted P(Y) code



The Global Positioning System

Modulation Scheme

- Quadrature BPSK modulation



Ref: JNC 2010 GPS 101 Short Course by Jacob Campbell

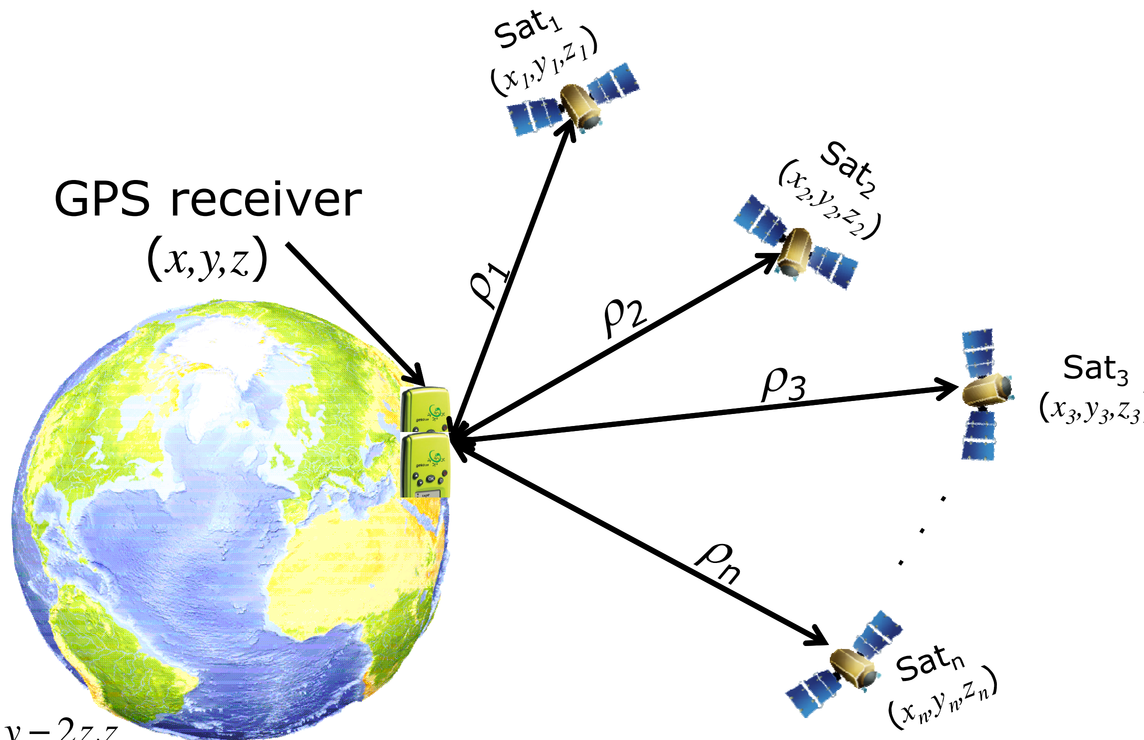
The Global Positioning System
Signal Processing

- Code and Carrier Phase Processing
  - Code used to determine user’s gross position
  - Carrier phase difference can be used to gain more accurate position
    - Timing of signals must be known to within one carrier cycle

GPS Measurement Equations

Pseudorange

All measurements in ECEF coordinates



$$\rho_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2}$$

$$\rho_i^2 = x_i^2 + x^2 - 2x_i x + y_i^2 + y^2 - 2y_i y + z_i^2 + z^2 - 2z_i z$$

$$\rho_i^2 - \left(x_i^2 + y_i^2 + z_i^2\right) - \left(x^2 + y^2 + z^2\right) = -2x_i x - 2y_i y - 2z_i z$$

$$\begin{bmatrix} \rho_1^2 - \left(x_1^2 + y_1^2 + z_1^2\right) - r_e \\ \rho_2^2 - \left(x_2^2 + y_2^2 + z_2^2\right) - r_e \\ \vdots \\ \rho_n^2 - \left(x_n^2 + y_n^2 + z_n^2\right) - r_e \end{bmatrix} \simeq \begin{bmatrix} -2x_1 & -2y_1 & -2z_1 \\ -2x_2 & -2y_2 & -2z_2 \\ & \vdots & \\ -2x_n & -2y_n & -2z_n \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

ρ - pseudorange
r_e - Earth's radius

GPS Measurement Equations

Pseudorange

- A more realistic model is

$$\rho_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} + c\Delta T + n_i$$

- Can perturb this model to form

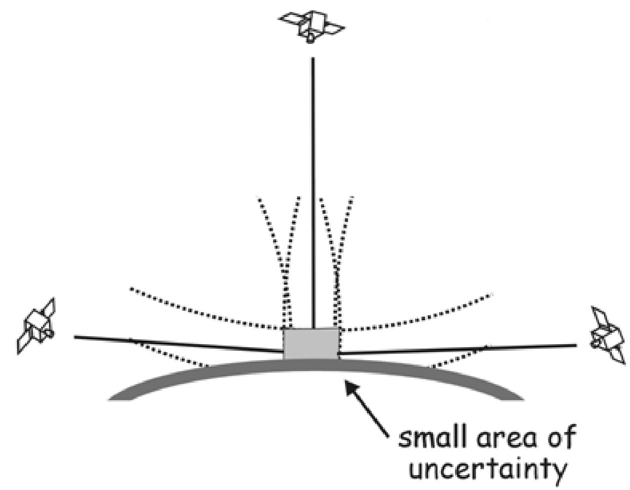
$$\begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \vdots \\ \Delta\rho_n \end{bmatrix} = \begin{bmatrix} & & & 1 \\ & & & 1 \\ \vdots & \vdots & \vdots & \vdots \\ & & & 1 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ c\Delta t \end{bmatrix} + \vec{n}$$

- This can be solved via least-squares or Kalman filter

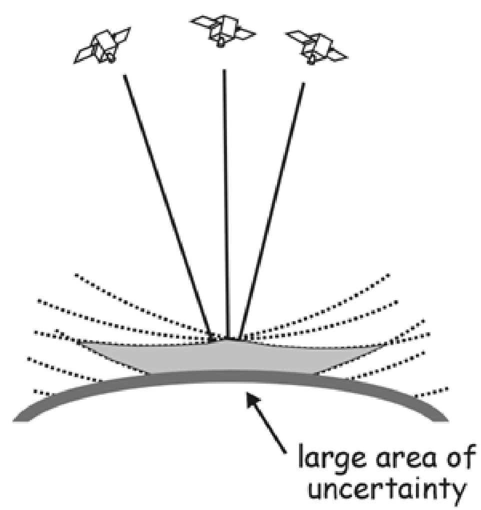
GPS Measurement Equations

Sources Of Error (GDOP)

- Geometry of satellite constellation wrt to receiver
- Good GDOP occurs when
  - Satellites just above the horizon spaced and one satellite directly overhead



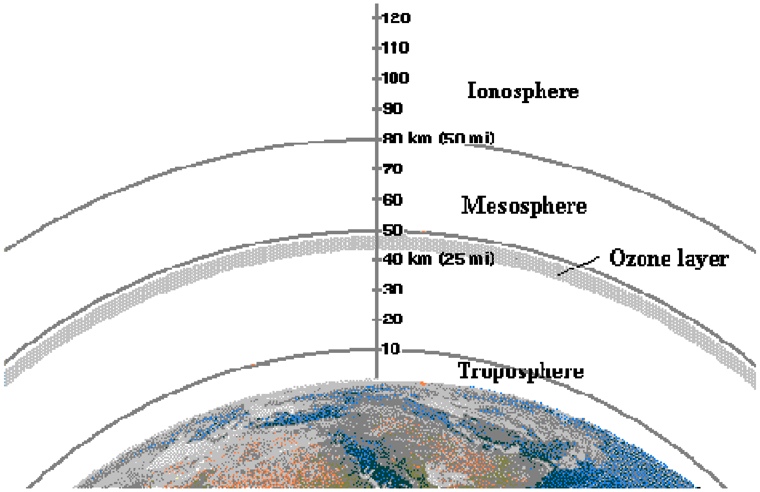
- Bad GDOP when pseudurange vectors are almost linearly dependent

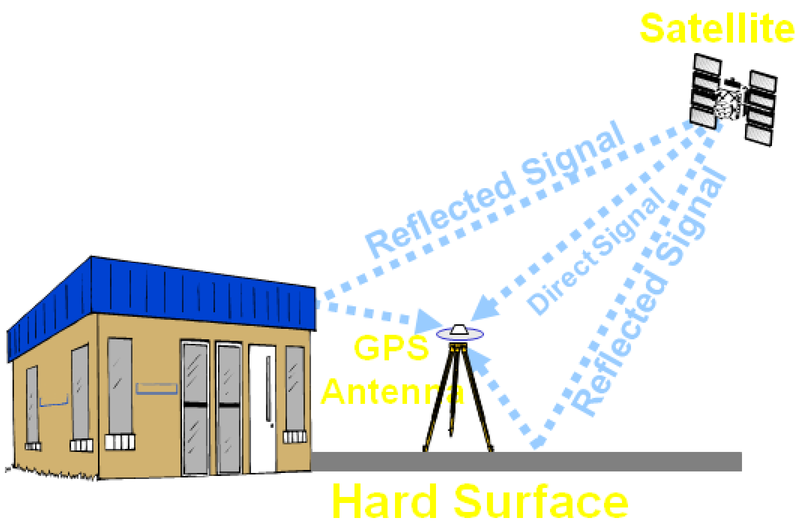


GPS Measurement Equations

Sources Of Error

- Selective Availability
  - Intentional errors in PRN
  - Discontinued in 5/1/2000
- Atmospheric Effects
  - Ionospheric
  - Tropospheric
- Multipath
- Ephemeris Error (satellite position data)
- Satellite Clock Error
- Receiver Clock Error





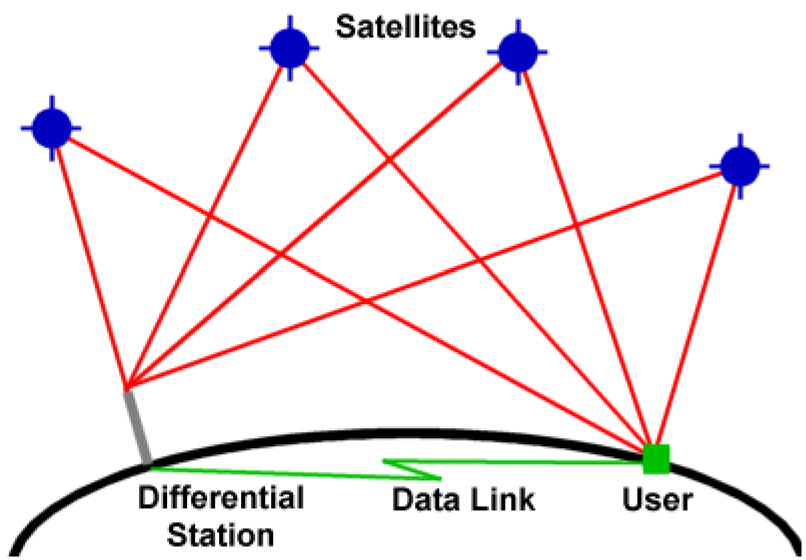
GPS Measurement Equations
Error Mitigation Techniques

- Carriers at L1 and L2 frequencies
  - Ionospheric error is frequency dependent so using two frequencies helps to limit error
- Differential GPS
  - Post-Process user measurements using measured error values
- Space Based Augmentation Systems(SBAS)
  - Examples are U.S. Wide Area Augmentation System (WAAS), European Geostationary Navigational Overlay Service (EGNOS)
  - SBAS provides atmospheric, ephemeris and satellite clock error correction values in real time

GPS Measurement Equations

Error Mitigation Techniques - Differential GPS

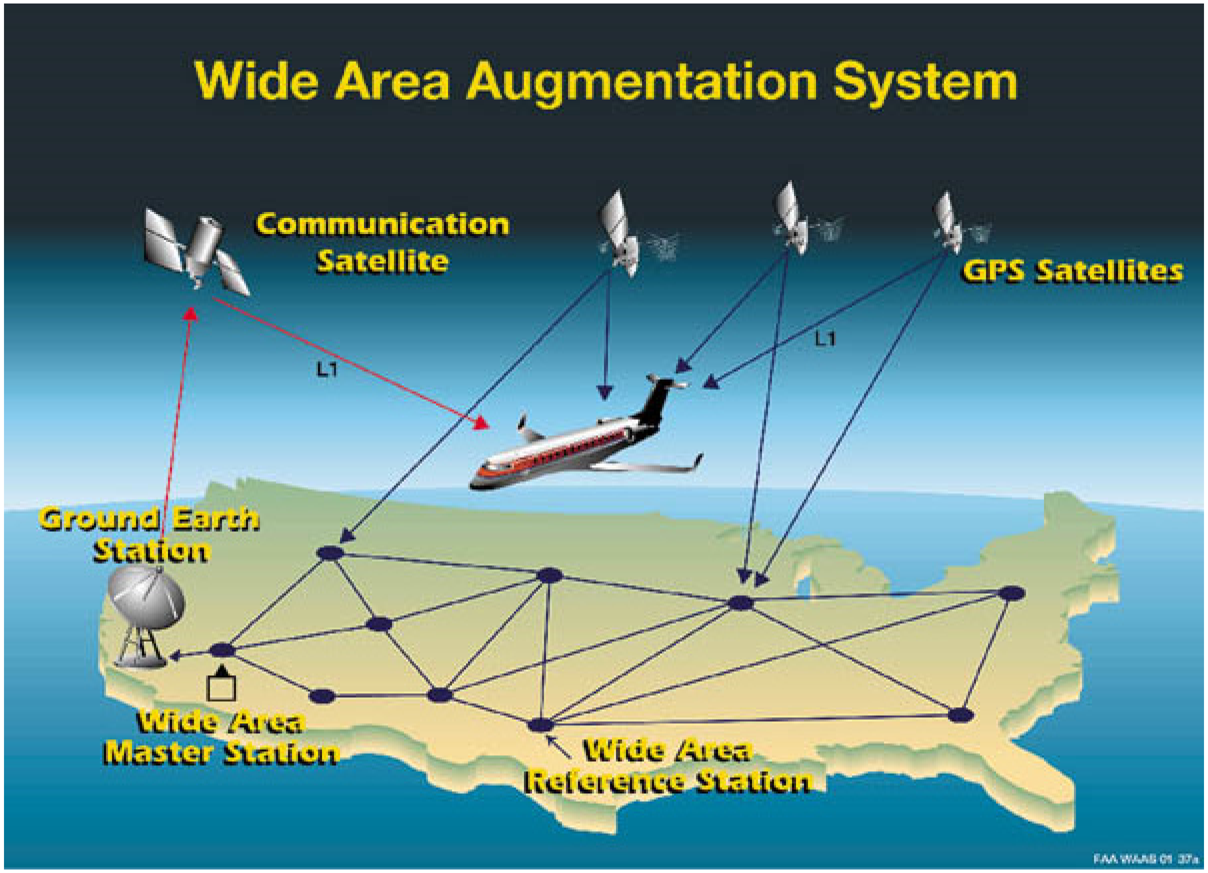
- Uses a GPS receiver at a fixed, surveyed location to measure error in pseudorange signals from satellites
  - Pseudorange error for each satellite is subtracted from mobile receiver before calculating position (typically post processed)



GPS Measurement Equations

Error Mitigation Techniques - WAAS/EGNOS

- Provide corrections based on user position
- Assumes atmospheric error is locally correlated



GPS Measurement Equations

Sources Of Error

- Single satellite pseudorange measurement

$$\rho_{measured} = \rho_{true} + e_{SA} + e_{ionospheric} + e_{tropospheric} + e_{ephemeris} + e_{satellite\ clock} + e_{receiver\ clock} + e_{multipath}$$

- GPS error summary

L1 C/A (with S/A off)

Error Source	Bias	Random	Total	DGPS
Ephemeris	2.1	0	2.1	0
Satellite Clock	1	0.7	2.1	0
Ionosphere	4	0.5	4	0.4
Troposphere	0.5	0.5	0.7	0.2
Multipath	1	1	1.4	1.4
Receiver measurement	0.5	0.2	0.5	0.5
User Equiv Range Error	5.1	1.4	5.3	1.6
Vertical 1 σ error VDOP			12.8	3.9
Horizontal 1 σ error HDOP 2.0			10.2	3.1

PPS Dual Frequency P/Y Code

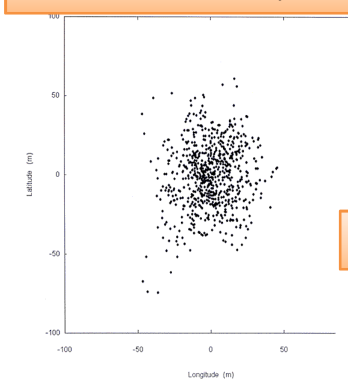
Error Source	Bias	Random	Total	DGPS
Ephemeris	2.1	0	2.1	0
Satellite Clock	2.0	0.7	2.1	0
Ionosphere	1.0	0.5	1.2	0.1
Troposphere	1.0	1.0	1.4	1.4
Multipath	1	1	1.4	1.4
Receiver measurement	0.5	0.2	0.5	0.5
User Equiv Range Error	3.3	1.5	3.6	1.5
Vertical 1 σ error VDOP			8.6	3.7
Horizontal 1 σ error HDOP 2.0			6.6	3.0

Ref: Navigation System Design by Eduardo Nebot, Centre of Excellence for Autonomous Systems, The University of Sydney

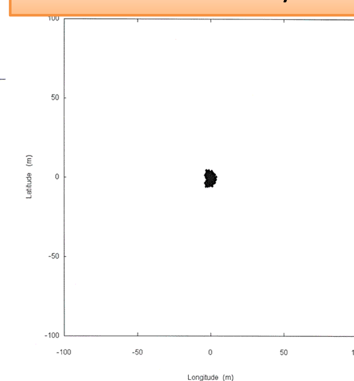
GPS Accuracy - PPS

PPS	CEP/50 %	DRMS	2DRMS/95%
Position			
Horizontal	8 m	10.5 m	21 m
Vertical	9 m	14 m	28 m
Spherical	*16 m*	18 m	36 m
Velocity			
Any Axis	0.07 m/sec	*0.1 m/sec*	0.2 m/sec
Time			
GPS	17 nsec	26 nsec	52 nsec

Selective Availability on



Selective Availability off



GPS Future

	II/IIA
Number SV's	28
First Launch	1989
Satellite Weight (Kg)	900
Power (W)	1,100
Design Life (Years)	7.5
Unit Cost ($M)	43
In Use (2010)	11

Ref: JNC 2010 GPS 101 Short Course by Jacob Campbell

GPS References

- References:
  - A compact introduction to GPS by The U. of Kentucky
    - www.ca.uky.edu/agc/pubs/aen/aen88/aen88.pdf
  - Principles and Practice of GPS Surveying (UNSW)
    - www.gmat.unsw.edu.au/snap/gps/gps_survey/principles_gps.htm
  - The Global Positioning Systems (GPS) Resource Library
    - http://www.gpsy.com/gpsinfo/