

Kinematic Equations of Linear and Rotational Motion (here motion of frame {2} wrt/frame {0} using an intermediate frame {1}):

$$C_2^0 = C_1^0 C_2^1 \quad (1)$$

$$\vec{\omega}_{02}^0 = \vec{\omega}_{01}^0 + C_1^0 \vec{\omega}_{12}^1 \quad (2)$$

$$\dot{\vec{\omega}}_{02}^0 = \dot{\vec{\omega}}_{01}^0 + [\vec{\omega}_{01}^0 \times] C_1^0 \vec{\omega}_{12}^1 + C_1^0 \dot{\vec{\omega}}_{12}^1 \quad (3)$$

$$\vec{r}_{02}^0 = \vec{r}_{01}^0 + C_1^0 \vec{r}_{12}^1 \quad (4)$$

$$\dot{\vec{r}}_{02}^0 = \dot{\vec{r}}_{01}^0 + [\vec{\omega}_{01}^0 \times] C_1^0 \vec{r}_{12}^1 + C_1^0 \dot{\vec{r}}_{12}^1 \quad (5)$$

$$\ddot{\vec{r}}_{02}^0 = \ddot{\vec{r}}_{01}^0 + [\dot{\vec{\omega}}_{01}^0 \times] C_1^0 \vec{r}_{12}^1 + [\vec{\omega}_{01}^0 \times] [\vec{\omega}_{01}^0 \times] C_1^0 \vec{r}_{12}^1 + 2 [\vec{\omega}_{01}^0 \times] C_1^0 \dot{\vec{r}}_{12}^1 + C_1^0 \ddot{\vec{r}}_{12}^1 \quad (6)$$

where

$$\vec{\omega}_{02}^0 = \dot{\theta} C_2^0 \vec{\rho}^2 = \dot{\theta} \vec{\rho}^0 = \omega_{02} \vec{\rho}^0 = \begin{bmatrix} \omega_{02x}^0 \\ \omega_{02y}^0 \\ \omega_{02z}^0 \end{bmatrix} \quad (7)$$

$$[\vec{\omega}_{02}^0 \times] = \begin{bmatrix} 0 & -\omega_{02z}^0 & \omega_{02y}^0 \\ \omega_{02z}^0 & 0 & -\omega_{02x}^0 \\ -\omega_{02y}^0 & \omega_{02x}^0 & 0 \end{bmatrix} \quad (8)$$

Relationship Between ECI Frame {i} and ECEF Frame {e}:

$$\vec{0} = \vec{r}_{ie}^i = \dot{\vec{r}}_{ie}^i = \ddot{\vec{r}}_{ie}^i = \dot{\vec{\omega}}_{ie}^i \quad (9)$$

$$\vec{\omega}_{ie}^i = \begin{bmatrix} 0 \\ 0 \\ \omega_{ie} \end{bmatrix} \quad (10)$$

$$C_e^i = \begin{bmatrix} \cos(\omega_{ie}(t - t_0)) & -\sin(\omega_{ie}(t - t_0)) & 0 \\ \sin(\omega_{ie}(t - t_0)) & \cos(\omega_{ie}(t - t_0)) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (11)$$

Relationship between Curvilinear Motion of Locally-level Navigation Frame $\{n\}$ at Geodetic Height h_n above Surface of Ellipsoid and ECEF Frame $\{e\}$:

$$C_n^e = \begin{bmatrix} -\sin(L_n) \cos(\lambda_n) & -\sin(\lambda_n) & -\cos(L_n) \cos(\lambda_n) \\ -\sin(L_n) \sin(\lambda_n) & \cos(\lambda_n) & -\cos(L_n) \sin(\lambda_n) \\ \cos(L_n) & 0 & -\sin(L_n) \end{bmatrix} \quad (12)$$

$$\vec{r}_{en}^e = \begin{bmatrix} x_{en}^e \\ y_{en}^e \\ z_{en}^e \end{bmatrix} = \begin{bmatrix} (R_E + h_n) \cos(L_n) \cos(\lambda_n) \\ (R_E + h_n) \cos(L_n) \sin(\lambda_n) \\ ((1 - e^2)R_E + h_n) \sin(L_n) \end{bmatrix} \quad (13)$$

$$\dot{\vec{r}}_{en}^e = C_n^e \underbrace{\begin{bmatrix} (R_N + h_n) \dot{L}_n \\ (R_E + h_n) \cos(L_n) \dot{\lambda}_n \\ -\dot{h}_n \end{bmatrix}}_{\dot{\vec{r}}_{en}^n} \quad (14)$$

$$\dot{\vec{r}}_{en}^n = \begin{bmatrix} v_{enx}^n \\ v_{eny}^n \\ v_{enz}^n \end{bmatrix} = \begin{bmatrix} (R_N + h_n) \dot{L}_n \\ (R_E + h_n) \cos(L_n) \dot{\lambda}_n \\ -\dot{h}_n \end{bmatrix} \quad (15)$$

$$\dot{\vec{p}}_n = \begin{bmatrix} \dot{L}_n \\ \dot{\lambda}_n \\ \dot{h}_n \end{bmatrix} = \begin{bmatrix} \frac{v_{enx}^n}{R_N + h_n} \\ \frac{v_{eny}^n}{(R_E + h_n) \cos(L_n)} \\ -v_{enz}^n \end{bmatrix} \quad (16)$$

where L_n , λ_n , h_n are geodetic latitude, longitude, and geodetic height of o^n , respectively, and median radius of curvature R_N and transverse radius of curvature R_E are given by

$$R_N = \frac{R_0(1 - e^2)}{(1 - e^2 \sin^2(L_n))^{3/2}} \quad (17)$$

$$R_E = \frac{R_0}{\sqrt{1 - e^2 \sin^2(L_n)}} \quad (18)$$