Lecture

INS/GPS Integration

EE 570: Location and Navigation

Lecture Notes Update on May 1, 2011

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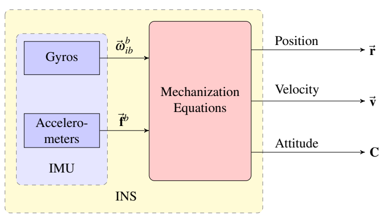
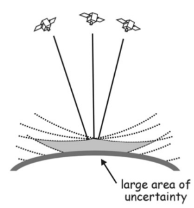
1 Overview

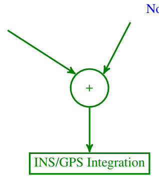
Need for Integration





Advantages	Disadvantages
Immune to RF Jaming	Drifts
High data rate	Errors are time dependent
High accuracy in short term	Need Initialization

Advantages	Disadvantages
Erros time-indep.	Sensitive to RF Interference
No initialization	No attitude information



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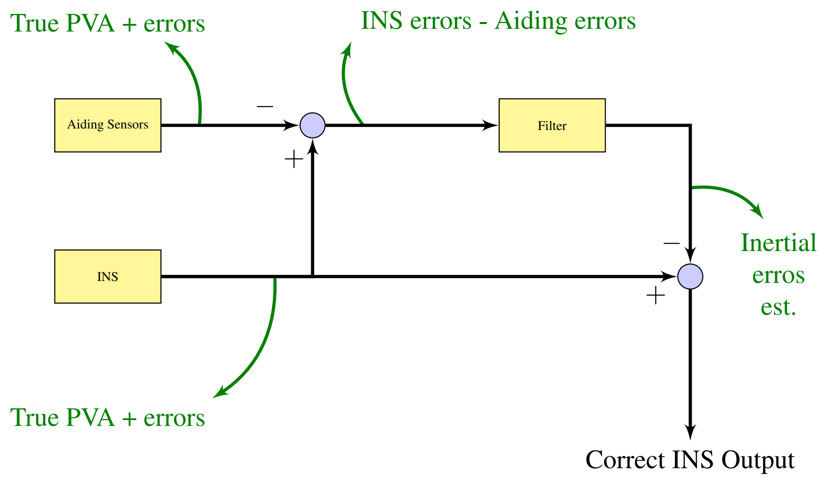
2 Integration Architectures

Open-Loop Integration



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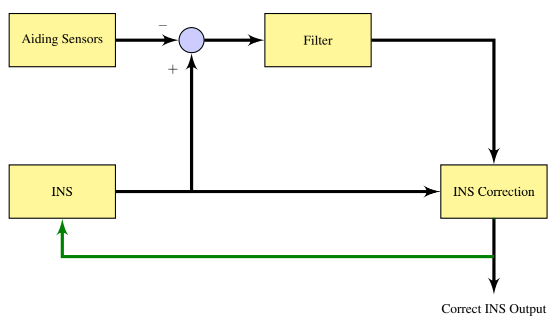
Closed-Loop Integration

If error estimates are fedback to correct the INS mechanization, a reset of the state estimates becomes necessary.



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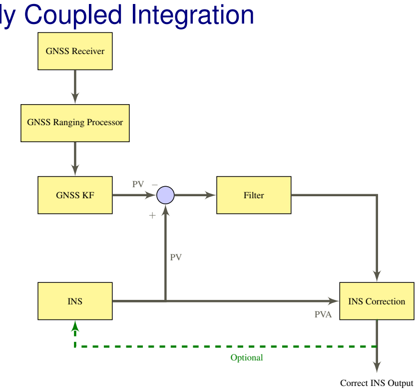
Loosely Coupled Integration



- Simple
- Cascade KF therefore integration KF BW must be less than that of GNSS KF (e.g. update interval of 10s)
- Minimum of 4 satellites required

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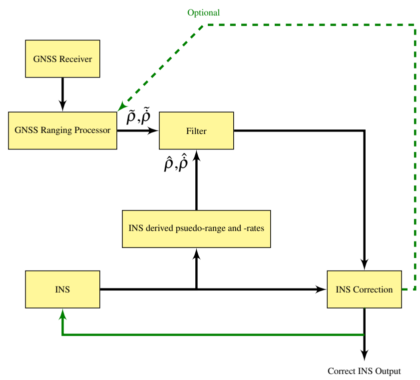
Tightly Coupled Integration



- No cascade KF
- KF BW must be kept less than the GNSS tracking loop
- Does not require 4 satellites

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INS Derived Psuedo-Range and -Rates

$$\hat{\rho}_{Cj,k} = \sqrt{[\hat{\vec{\mathbf{r}}}^e_{esj}(\tilde{t}_{st,j,k}) - \hat{\vec{\mathbf{r}}}^e_{ea,kj}]^T [\hat{\vec{\mathbf{r}}}^e_{esj}(\tilde{t}_{st,j,k}) - \hat{\vec{\mathbf{r}}}^e_{ea,kj}]} + \delta\hat{\rho}_{rc,k} + \delta\rho_{ie,j} \tag{1}$$

$$\hat{\dot{\rho}}_{Cj,k} = \hat{\vec{\mathbf{u}}}^{eT}_{as,j,k}[\hat{\vec{\mathbf{v}}}^e_{esj}(\tilde{t}_{st,j,k}) - \hat{\vec{\mathbf{v}}}^e_{ea,kj}]^T + \delta\hat{\dot{\rho}}_{rc,k} + \delta\dot{\rho}_{ie,j} \tag{2}$$

where

$$\vec{\mathbf{u}}^e_{as,j} = \frac{\vec{\mathbf{r}}^e_{es,j}(t_{st,j}) - \vec{\mathbf{r}}^a_{as,j}(t_{sa})}{\|\vec{\mathbf{r}}^e_{es,j}(t_{st,j}) - \vec{\mathbf{r}}^a_{as,j}(t_{sa})\|} \tag{3}$$

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3 Observability

Observability

- Attitude and acceleration errors are observable through growth in velocity and position errors.
- In level accelertion, heading error only produces velocity error, therefore requires significant maneuvering.
- If level and not accelerating, vertical accel bias is the only cause of vertical velocity error growth.

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4 Error Mechanization

ECEF Error Mechanization (loosely coupled)

Assuming errors are due to biases that are modeled as WGN.

$$\begin{pmatrix} \delta\dot{\vec{\psi}}^e_{eb} \\ \delta\dot{\vec{\mathbf{v}}}^e_{eb} \\ \delta\dot{\vec{\mathbf{r}}}^e_{eb} \\ \dot{\vec{\mathbf{b}}}_a \\ \dot{\vec{\mathbf{b}}}_g \end{pmatrix} = \mathbf{F}(t) \begin{pmatrix} \delta\vec{\psi}^e_{eb} \\ \delta\vec{\mathbf{v}}^e_{eb} \\ \delta\vec{\mathbf{r}}^e_{eb} \\ \vec{\mathbf{b}}_a \\ \vec{\mathbf{b}}_g \end{pmatrix} = \mathbf{F}(t)\vec{\mathbf{x}}(t) \tag{4}$$

where

$$\mathbf{F}(t) = \begin{pmatrix} -[\vec{\omega}^e_{ie}\times] & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & \mathbf{C}^e_b \\ -[\mathbf{C}^e_b\vec{\mathbf{f}}^b_{ib}\times] & -2\Omega^i_{ie} & \frac{2g_0}{\|\vec{\mathbf{r}}^e_{eb}\| r^e_{eS}}[\vec{\mathbf{r}}^e_{eb}(\vec{\mathbf{r}}^e_{eb})^T]\delta\vec{\mathbf{r}}^e_{eb} & \mathbf{C}^e_b & 0_{3\times3} \\ 0_{3\times3} & \mathbf{I}_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{pmatrix}$$

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5 Integration Filter

Kalman Filter

$$\hat{\vec{\mathbf{x}}}_{k|k-1} = \mathbf{\Phi}_{k-1}\hat{\vec{\mathbf{x}}}_{k-1|k-1} \tag{5}$$

$$\mathbf{P}_{k|k-1} = \mathbf{Q}_{k-1} + \mathbf{\Phi}_{k-1}\mathbf{P}_{k-1|k-1}\mathbf{\Phi}_{k-1}^T \tag{6}$$

$$\hat{\vec{\mathbf{x}}}_{k|k} = \hat{\vec{\mathbf{x}}}_{k|k-1} + \mathbf{K}_k(\vec{\mathbf{z}}_k - \mathbf{H}_k\hat{\vec{\mathbf{x}}}_{k|k-1}) \tag{7}$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\,\mathbf{P}_{k|k-1}\,(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)^T + \mathbf{K}_k\mathbf{R}_k\mathbf{K}_k^T \tag{8}$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^T\,(\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}_k)^{-1} \tag{9}$$

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Closed-Loop Kalman Filter

Since the errors are being fedback to correct the INS, the state estimate must be reset after each INS correction.

$$\hat{\vec{\mathbf{x}}}_{k|k-1} = 0 \tag{10}$$

$$\mathbf{P}_{k|k-1} = \mathbf{Q}_{k-1} + \mathbf{\Phi}_{k-1}\mathbf{P}_{k-1|k-1}\mathbf{\Phi}_{k-1}^T \tag{11}$$

$$\hat{\vec{\mathbf{x}}}_{k|k} = \mathbf{K}_k\vec{\mathbf{z}}_k \tag{12}$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\,\mathbf{P}_{k|k-1}\,(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)^T + \mathbf{K}_k\mathbf{R}_k\mathbf{K}_k^T \tag{13}$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^T\,(\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}_k)^{-1} \tag{14}$$

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Discretization

$$\mathbf{\Phi}_{k-1} \approx \mathbf{I} + \mathbf{F}\tau_s \tag{15}$$

$$\mathbf{Q} = \begin{pmatrix} n_{rg}^2\mathbf{I}_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & n_{ag}^2\mathbf{I}_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & n_{bad}^2\mathbf{I}_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & n_{bgd}^2\mathbf{I}_{3\times3} \end{pmatrix}\tau_s \tag{16}$$

where τ_s is the sample time, n_{rg}^2, n_{ag}^2, n_{bad}^2, n_{bgd}^2 are the PSD of the gyro and accel random noise, and accel and gyro bias variation, respectively. .12

6 Measurement Models

ECEF INS/GNSS Loosely Coupled

$$\vec{\mathbf{z}}_k^e = \begin{pmatrix} \tilde{\vec{\mathbf{r}}}_{GPS} - \hat{\vec{\mathbf{r}}}_{eb}^e \\ \tilde{\vec{\mathbf{v}}}_{GPS} - \hat{\vec{\mathbf{v}}}_{eb}^e \end{pmatrix} \tag{17}$$

$$\mathbf{H} = \begin{pmatrix} 0_{3\times3} & 0_{3\times3} & -\mathbf{I}_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & -\mathbf{I}_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{pmatrix} \tag{18}$$

Theoretically, the lever arm from the INS to the GNSS antenna needs to be included, but in practice, the coupling of the attitude errors and gyro biases into the measurement through the lever arm is week. .13

ECEF INS/GNSS Tightly Coupled

Pseudo-ranges are used instead of XYZ.

$$\vec{\mathbf{z}} = \begin{pmatrix} \vec{\mathbf{z}}_\rho \\ \vec{\mathbf{z}}_{\dot{\rho}} \end{pmatrix} \tag{19}$$

where

$$\vec{\mathbf{z}}_\rho = (\tilde{\rho}_{C1} - \hat{\rho}_{C1}, \tilde{\rho}_{C2} - \hat{\rho}_{C2}, \ldots, \tilde{\rho}_{Cn} - \hat{\rho}_{Cn}) \tag{20}$$

$$\vec{\mathbf{z}}_{\dot{\rho}} = (\tilde{\dot{\rho}}_{C1} - \hat{\dot{\rho}}_{C1}, \tilde{\dot{\rho}}_{C2} - \hat{\dot{\rho}}_{C2}, \ldots, \tilde{\dot{\rho}}_{Cn} - \hat{\dot{\rho}}_{Cn}) \tag{21}$$

$$\vec{\mathbf{x}}(t) = \begin{pmatrix} \delta\vec{\psi}^e_{eb} & \delta\vec{\mathbf{v}}^e_{eb} & \delta\vec{\mathbf{r}}^e_{eb} & \vec{\mathbf{b}}_a & \vec{\mathbf{b}}_g & \delta\rho_{rc} & \delta\dot{\rho}_{rc} \end{pmatrix}^T \tag{22}$$

$\tilde{\rho}_{Cj}$, and $\tilde{\dot{\rho}}_{Cj}$ and $\hat{\rho}_{Cj}$, and $\hat{\dot{\rho}}_{Cj}$ are the psuedo-ranges and rates obtained from the GNSS and INS, respectively, for the jth satallite. These equations are none linear and an EKF needs to be used. $\delta\rho_{rc}$ and $\delta\dot{\rho}_{rc}$ are the clock bias and drift. .14

Tightly Coupled Linearized Measurement Matrix

$$\mathbf{H}^e = \left(\begin{array}{ccccccc} 0_{1\times3} & 0_{1\times3} & \vec{\mathbf{u}}^{eT}_{as,1} & 0_{1\times3} & 0_{1\times3} & 1 & 0_{1\times3} \\ 0_{1\times3} & 0_{1\times3} & \vec{\mathbf{u}}^{eT}_{as,2} & 0_{1\times3} & 0_{1\times3} & 1 & 0_{1\times3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0_{1\times3} & 0_{1\times3} & \vec{\mathbf{u}}^{eT}_{as,n} & 0_{1\times3} & 0_{1\times3} & 1 & 0_{1\times3} \\ \hline 0_{1\times3} & \vec{\mathbf{u}}^{eT}_{as,1} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 1 \\ 0_{1\times3} & \vec{\mathbf{u}}^{eT}_{as,2} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0_{1\times3} & \vec{\mathbf{u}}^{eT}_{as,n} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 1 \end{array}\right) \tag{23}$$

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