Lecture
Gyro and Accel Noise Characteristics
EE 570: Location and Navigation

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1 Allan Variance

Allan Variance

1. Divide your N-point data sequence into adjacent windows of size $n = 1, 2, 4, 8, \ldots, M \leq N/2$.
2. For every n generate the sequence

$$y_j(n) = \frac{x_{nj} + x_{nj+1} + \cdots + x_{nj+n-1}}{n}, \quad j = 0, 1, \ldots, \left[\frac{N}{n}\right] - 1 \tag{1}$$

3. Plot log-log of the Allan deviation which is square root of

$$\sigma^2_{Allan}(nT_s) = \frac{1}{2(N-1)} \sum_{j=1}^{N-1} (y_j - y_{j-1})^2 \tag{2}$$

versus averaging time $\tau = nT_s$

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2 Gyro Noise Characteristics

Gyro Constant Bias ($^\circ/h$)

A constant in the output of a gyro in the absence of rotation, in $^\circ/h$.

Error Growth
Linearly growing error in the angle domain of εt.

Model
Random constant.

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Gyro Integrated White Noise

Assuming the rectangular rule is used for integration, a sampling period of T_s and a time span of nT_s.

$$\int_0^t \varepsilon(\tau)d\tau = T_s \sum_{i=1}^{n} \varepsilon(t_i) \tag{3}$$

since $\mathbb{E}[\varepsilon(t_i)] = 0$ and $Cov(\varepsilon(t_i), \varepsilon(t_j)) = 0$ for all $i \neq j$, $Var[\varepsilon(t_i)] = \sigma^2$

$$\mathbb{E}\left[\int_0^t \varepsilon(\tau)d\tau\right] = T_s n \mathbb{E}[\varepsilon(t_i)] = 0, \forall i \tag{4}$$

$$Var\left[\int_0^t \varepsilon(\tau)d\tau\right] = T_s^2 n Var[\varepsilon(t_i)] = T_s t \sigma^2, \forall i \tag{5}$$

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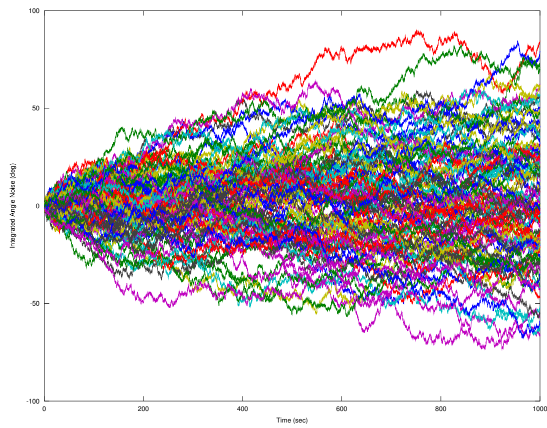
Gyro Integrated White Noise



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Angle Random Walk ($^\circ/\sqrt{h}$)

Integrated noise resulted in zero-mean random walk with standard deviation that grows with time as

$$\sigma_\theta = \sigma\sqrt{T_s t} \tag{6}$$

We define *ARW* as

$$ARW = \sigma_\theta(1) \qquad (^\circ/\sqrt{h}) \tag{7}$$

In terms of PSD

$$ARW(^\circ/\sqrt{h}) = \frac{1}{60}\sqrt{PSD((^\circ/h)^2/Hz)} \tag{8}$$

Error Growth
ARW times root of the time in hours.

Model
White noise.

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Gyro Bias Instability ($^\circ/h$)

- Due to flicker noise with spectrum $1/F$.
- Results in random variation in the bias.
- Normally more noticeable at low frequencies.
- At high frequencies, white noise is more dominant.

Error Growth
Variance grows over time.

Model
First order Gauss-Markov.

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3 Accel Noise Characteristics

Accel Constant Bias (μg)

A constant deviation in the accelerometer from the true value, in m/s^2.

Error growth
Double integrating a constant bias error of ε results in a quadratically growing error in position of $\varepsilon t^2/2$.

Model
Random constant.

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Velocity Random Walk ($m/s/\sqrt{h}$)

Integrating accelerometer output containing white noise results in velocity random walk (VRW) $(m/s/\sqrt{h})$. Similar to development of ARW, if we double integrate white noise we get

$$\iint_0^t \varepsilon(\tau)d\tau d\tau = T_{s,sensor}^2 \sum_{i=1}^{n} \sum_{j=1}^{i} \varepsilon(t_j) \tag{9}$$

Error Growth
Computing the variance results in

$$\sigma_p \approx \sigma t^{(3/2)} \sqrt{\frac{T_s}{3}} \tag{10}$$

Model
White noise.

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Accel Bias Stability (μg)

Error growth
Grows as $t^{5/2}$.

Model
First order Gauss-Markov.

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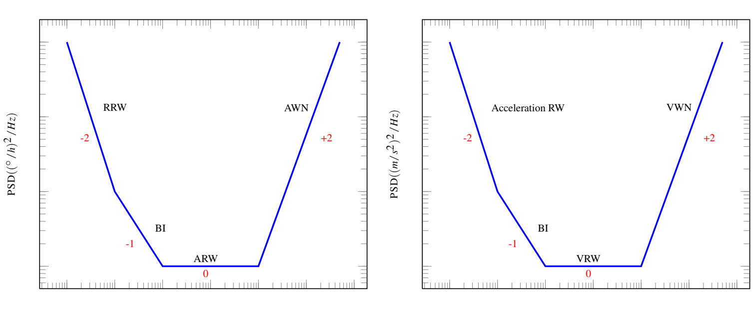
4 Using PSD and Allan Variance

One-sided PSD - Typical Slopes for rate and acceleration data



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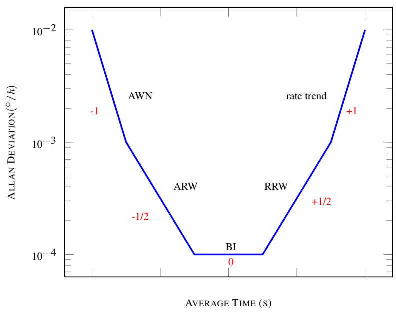
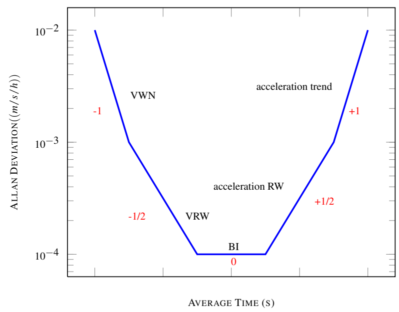
Allan Deviation - Typical Slopes for rate and acceleration data





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Noise Parameters

Noise Type	AV $\sigma^2(\tau)$	PSD (2-sided)
Quantization Noise	$3\frac{\alpha^2}{\tau^2}$	$(2\pi f)^2\alpha^2 T_s$
Angle/Velocity Random Walk	$\frac{\alpha^2}{\tau}$	α^2
Flicker Noise	$\frac{2\alpha^2 \ln(2)}{\pi}$	$\frac{\alpha^2}{2\pi f}$
Angular Rate/Accel Random Walk	$\frac{\alpha^2 \tau}{3}$	$\frac{\alpha^2}{(2\pi f)^2}$
Ramp Noise	$\frac{\alpha^2 \tau^2}{2}$	$\frac{\alpha^2}{(2\pi f)^3}$

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