EE 570: Location and Navigation

INS/GPS Integration

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Open-Loop Integration

Integration Architectures
Aly El-Osery (NMT)

Measurement Models
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Correct INS Output
Open-Loop Integration

Aiding Sensors + Filter

INS

True PVA + errors

Correct INS Output
Open-Loop Integration

Aiding Sensors + True PVA + errors → Filter

Correct INS Output

INS

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Measurement Models
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Other Considerations
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Open-Loop Integration

True PVA + errors

Aiding Sensors

Aiding errors - INS errors

INS

Filter

+ 

\[ \text{Correct INS Output} \]

Integration Architectures

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Measurement Models

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Other Considerations

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Open-Loop Integration

Aiding Sensors

True PVA + errors

Aiding errors - INS errors

INS

Filter

Inertial errors est.

Correct INS Output

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Other Considerations

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If error estimates are fed back to correct the INS mechanization, a reset of the state estimates becomes necessary.
Loosely Coupled Integration

- Simple
- Cascade KF therefore integration KF BW must be less than that of GNSS KF (e.g. update interval of 10s)
- Minimum of 4 satellites required

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Loosely Coupled Integration

- Simple
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- Minimum of 4 satellites required
Tightly Coupled Integration

- No cascade KF
- KF BW must be kept less than the GNSS tracking loop
- Does not require 4 satellites
Tightly Coupled Integration

- No cascade KF
- KF BW must be kept less than the GNSS tracking loop
- Does not require 4 satellites
Theoretically, the lever arm from the INS to the GNSS antenna needs to be included, but in practice, the coupling of the attitude errors and gyro biases into the measurement through the lever arm is week.

\[ \mathbf{z}_k^e = \begin{pmatrix} \tilde{\mathbf{r}}_{GPS} - \hat{\mathbf{r}}_{eb}^e \\ \tilde{\mathbf{v}}_{GPS} - \hat{\mathbf{v}}_{eb}^e \end{pmatrix} \]  

(1)

\[
\mathbf{H} = \begin{pmatrix}
0_{3 \times 3} & 0_{3 \times 3} & I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\
0_{3 \times 3} & I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3}
\end{pmatrix}
\]  

(2)
INS Derived Pseudo-Range and Rates

The computed INS pseudo-range and rates with respect to the $j$th satellite, $\hat{\rho}_j$ and $\dot{\hat{\rho}}_j$

$$\hat{\rho}_j = \sqrt{[\hat{\mathbf{r}}_{es,j} - \hat{\mathbf{r}}_{eb}]^T [\hat{\mathbf{r}}_{es,j} - \hat{\mathbf{r}}_{eb}]} + \delta \hat{\rho}_{rc,j}$$  \hspace{1cm} (3)

$$\dot{\hat{\rho}}_j = \bar{\mathbf{u}}_j^T [\hat{\mathbf{v}}_{s,j} - \hat{\mathbf{v}}_{eb}]^T + \delta \dot{\hat{\rho}}_{rc,j}$$  \hspace{1cm} (4)

where

$$\bar{\mathbf{u}}_j = \frac{\hat{\mathbf{r}}_{es,j} - \hat{\mathbf{r}}_{eb}}{\|\hat{\mathbf{r}}_{es,j} - \hat{\mathbf{r}}_{eb}\|}$$  \hspace{1cm} (5)
Pseudo-ranges are used instead of XYZ.

\[
\vec{z} = \begin{pmatrix} \vec{z}_\rho \\ \vec{z}_\dot{\rho} \end{pmatrix}
\]

(6)

where

\[
\vec{z}_\rho = (\rho_{gps,1} - \hat{\rho}_1, \rho_{gps,2} - \hat{\rho}_2, \ldots, \rho_{gps,n} - \hat{\rho}_n)
\]

(7)

\[
\vec{z}_\dot{\rho} = (\dot{\rho}_{gps,1} - \dot{\hat{\rho}}_1, \dot{\rho}_{gps,2} - \dot{\hat{\rho}}_2, \ldots, \dot{\rho}_{gps,n} - \dot{\hat{\rho}}_n)
\]

(8)

\[
\vec{x}(t) = \begin{pmatrix} \delta \vec{y}_{eb}^e \\ \delta \vec{v}_{eb}^e \\ \delta \vec{r}_{eb}^e \\ \vec{b}_a \\ \vec{b}_g \\ \delta \rho_{rc} \\ \delta \dot{\rho}_{rc} \end{pmatrix}
\]

(9)

\(\rho_{gps,j}\), and \(\dot{\rho}_{gps,j}\) and \(\hat{\rho}_j\), and \(\dot{\hat{\rho}}_j\) are the pseudo-ranges and rates obtained from the GNSS and INS, respectively, for the jth satellite. These equations are none linear and an EKF needs to be used. \(\delta \rho_{rc}\) and \(\delta \dot{\rho}_{rc}\) are the clock bias and drift.
Tightly Coupled Linearized Measurement Matrix

\[
H = 
\begin{pmatrix}
0_{1\times3} & 0_{1\times3} & -\vec{u}_1^T & 0_{1\times3} & 0_{1\times3} & 1 & 0_{1\times3} \\
0_{1\times3} & 0_{1\times3} & -\vec{u}_2^T & 0_{1\times3} & 0_{1\times3} & 1 & 0_{1\times3} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0_{1\times3} & 0_{1\times3} & -\vec{u}_n^T & 0_{1\times3} & 0_{1\times3} & 1 & 0_{1\times3} \\
0_{1\times3} & -\vec{u}_1^T & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 1 & \vdots \\
0_{1\times3} & -\vec{u}_2^T & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 1 & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0_{1\times3} & -\vec{u}_n^T & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 1 & \vdots \\
\end{pmatrix}
\]
Are the states observable given a certain set of measurements? The system is observable if the observability matrix

\[
\mathcal{O}(k) = \begin{bmatrix}
H(k - n + 1) \\
H(k - n - 2)\Phi(k - n + 1) \\
\vdots \\
H(k)\Phi(k - 1) \ldots \Phi(k - n + 1)
\end{bmatrix}
\]

where \( n \) is the number of states, has a rank of \( n \). The rank of \( \mathcal{O} \) is a binary indicator and does not provide a measure of how close the system is to being unobservable, hence, is prone to numerical ill-conditioning.
In addition to the computation of the rank of $\mathcal{O}(k)$, compute the Singular Value Decomposition (SVD) of $\mathcal{O}(k)$ as

$$\mathcal{O} = U\Sigma V^*$$  \hspace{1cm} (12)

and observe the diagonal values of the matrix $\Sigma$. Using this approach it is possible to monitor the variations in the system observability due to changes in system dynamics.
In many cases the data from the aiding sensor may get corrupt and even not available. In those cases we can’t use those measurements in the Kalman filter. Therefore

1. Set the Kalman gain to zero, or
2. Do not run the state or error covariance update steps.