• As the name implies inertial sensors measure motion wrt an inertial frame
  ▪ Advantages: Self-contained & non-reliant on external fields (e.g. EM radiation, Earth’s magnetic field, ...)
  ▪ Disadvantages: Typically rate measurements & expensive

• Accelerometers measure linear acceleration
  ▪ Actually measure specific force, typically, in the body frame

\[
\ddot{f}_{ib} = \ddot{a}_{ib} - \gamma_{ib}
\]

• Gyroscopes measure angular velocity
  ▪ Most gyroscopes measure angular speed, typically, in the body frame

\[
\omega_{ib}^b
\]
Navigation Sensors and INS Mechanization

Inertial Sensors - Accelerometer

- Inertial Sensors
  - Accelerometers
  - Gyroscopes
  - Pendulous Mass
  - Vibratory
    - Closed loop
    - Open loop
  - Closed loop
  - Open loop
• The Pendulous Mass Accelerometer
  
  - A mass, a suspension system, and a sensing element
  - Displacement $\propto$ applied force resolved along the sensitive axis
  - Modeled as a basic 2\textsuperscript{nd} order system
    \[ f = m\ddot{x} + b\dot{x} + kx \]
  - In steady state $m\ddot{x} \approx -kx$
    hence $SF = \left( \frac{x}{\dot{x}} \right) = -\frac{m}{k}$
• Closed-loop version generates a force to null the displacement
  ▪ Can improve linearity and measurement range
• **Pendulous Accelerometer**
  - Closed loop configuration
    - Improved linearity

![Diagram of a pendulous accelerometer](image-url)

*Figure: Clipp (2006)*
• **Vibratory accelerometers**
  - Vibrating Beam Accelerometers (VBA)
  - Acceleration causes a change in resonance frequency

\[ f_0 = k \sqrt{Tension} \]
• MEMS Accelerometers

www.ett.bme.hu/memsedu
• MEMS Accelerometers
  - Spring and mass from silicon and add fingers make a variable differential capacitor
  - Change in displacement => change in capacitance
• Rotating Mass Gyros
  ▪ Conservation of Angular Momentum
  ▪ The spinning mass will resist change in its angular momentum
  ▪ Angular momentum
    ○ $H = I \omega$ (Inertia * Angular velocity)
  ▪ By placing the gyro in a pair of frictionless gimbals it is free to maintain its inertial spin axis
  ▪ By placing an index on the x-gimbal axes and y-gimbal axis two degrees of orientational motion can be measured
Rotating Mass Gyros

- **Precession**
  - Disk is spinning about z-axis
  - Apply a torque about the x-axis
  - Results in precession about the y-axis
  - \( \tau = \omega \times H \)
**Fiber Optical Gyro (FOG)**

- Basic idea is that light travels at a constant speed.
- If rotated (orthogonal to the plane) one path length becomes longer and the other shorter.
- This is known as the Sagnac effect.
- Measuring path length change (over a $dt$) allows $\omega$ to be measured.
**Fiber Optical Gyro (FOG)**

- Measure the time difference between the CW and CCW paths
- CW transit time = \( t_{CW} \)
- CCW transit time = \( t_{CCW} \)

\[
\begin{align*}
L_{CW} &= 2\pi R + R \omega t_{CW} = ct_{CW} \\
L_{CCW} &= 2\pi R - R \omega t_{CCW} = ct_{CCW} \\
t_{CW} &= \frac{2\pi R}{c - R \omega} \\
t_{CCW} &= \frac{2\pi R}{c + R \omega} \\
 \Rightarrow \Delta t &= \frac{4\pi R^2 \omega}{c^2} \\
\Delta t &\approx \frac{N 4A \omega}{c^2} \\
\phi_c &\approx 2\pi \Delta f_c = 2\pi \Delta t c / \lambda_0 = \frac{8\pi NA \omega}{c \lambda_0}
\end{align*}
\]
• **Ring Laser Gyro**
  
  - A helium-neon laser produces two light beams, one traveling in the CW direction and the other in the CCW direction
  
  - When rotating
    - The wavelength in dir of rotation increases (decrease in freq)
    - The wavelength in opposite dir decreases (increase in freq)
    - Similarly, it can be shown that
      \[
      \Delta f \approx \frac{4A\omega}{\lambda_0}
      \]
• **Vibratory Coriolis Angular Rate Sensor**
  - Virtually all MEMS gyros are based on this effect

\[ a_{\text{Coriolis}} = 2\omega \times v \]
• **Basic Planar Vibratory Gyro**
Navigation Sensors and INS Mechanization
Inertial Sensors – Gyroscopes: Coriolis Effect

• In plane sensing (left)
• Out of plane sensing (right)

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Navigation Sensors and INS Mechanization
Inertial Sensors - Summary

• **Accelerometers**
  - Measure specific force of the body frame wrt the inertial frame in the body frame coordinates
    - Need to subtract the acceleration due to gravity to obtain the motion induced quantity
  - In general, all points on a rigid body do **NOT** experience the same linear velocity

• **Gyroscopes**
  - Measure the inertial angular velocity
    - Essentially, the rate of change of orientation
  - All points on a rigid body experience the same angular velocity