EE 570: Location and Navigation

Gyro and Accel Noise Characteristics

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Gyro Constant Bias ($^\circ/h$)

A constant in the output of a gyro in the absence of rotation, in $^\circ/h$.

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Error Growth

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Model

Random constant.

Assuming the rectangular rule is used for integration, a sampling period of T_s and a time span of nT_s.

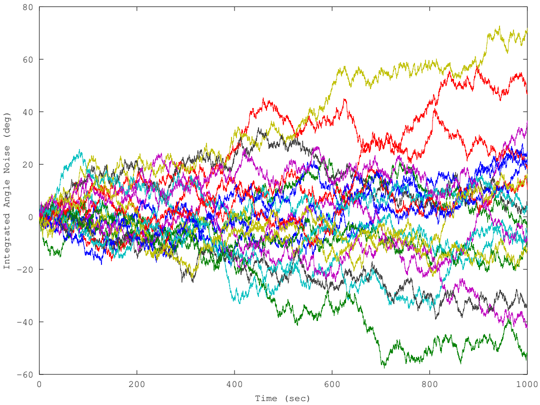
$$\int_0^t \epsilon(\tau)d\tau = T_s \sum_{i=1}^{n} \epsilon(t_i) \tag{1}$$

since $\mathbb{E}[\epsilon(t_i)] = 0$ and $Cov(\epsilon(t_i), \epsilon(t_j)) = 0$ for all $i \neq j$, $Var[\epsilon(t_i)] = \sigma^2$

$$\mathbb{E}\left[\int_0^t \epsilon(\tau)d\tau\right] = T_s n \mathbb{E}[\epsilon(t_i)] = 0, \forall i \tag{2}$$

$$Var\left[\int_0^t \epsilon(\tau)d\tau\right] = T_s^2 n Var[\epsilon(t_i)] = T_s t \sigma^2, \forall i \tag{3}$$

Gyro Integrated White Noise



Angle Random Walk ($^\circ/\sqrt{h}$)

Integrated noise resulted in zero-mean random walk with standard deviation that grows with time as

$$\sigma_\theta = \sigma\sqrt{T_s t} \tag{4}$$

We define *ARW* as

$$ARW = \sigma_\theta(1) \qquad (^\circ/\sqrt{h}) \tag{5}$$

In terms of PSD

$$ARW(^\circ/\sqrt{h}) = \frac{1}{60}\sqrt{PSD((^\circ/h)^2/Hz)} \tag{6}$$

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Error Growth

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Model

First order Gauss-Markov.

Accel Constant Bias (μg)

A constant deviation in the accelerometer from the true value, in m/s^2.

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Error growth

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Integrating accelerometer output containing white noise results in velocity random walk (VRW) $(m/s/\sqrt{h})$. Similar to development of ARW, if we double integrate white noise we get

$$\iint_0^t \epsilon(\tau)d\tau d\tau = T_{s,sensor}^2 \sum_{i=1}^{n}\sum_{j=1}^{i}\epsilon(t_j) \tag{7}$$

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Velocity Random Walk ($m/s/\sqrt{h}$)

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Error growth

Grows as $t^{5/2}$.

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Model

First order Gauss-Markov.

It is a time domain analysis techniques designed originally for characterizing noise in clocks. It was first proposed by David Allan in 1966.

Allan Variance Computation

1. Divide your N-point data sequence into adjacent windows of size $n = 1, 2, 4, 8, \ldots, M \leq N/2$.
2. For every n generate the sequence

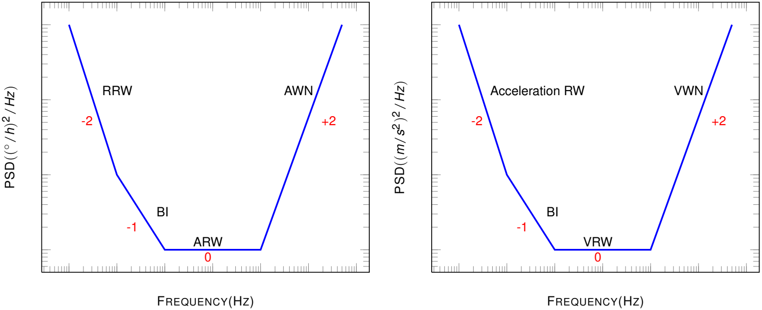
$$y_j(n) = \frac{x_{nj} + x_{nj+1} + \cdots + x_{nj+n-1}}{n}, \quad j = 0, 1, \ldots, \left\lfloor \frac{N}{n} \right\rfloor - 1 \quad (9)$$

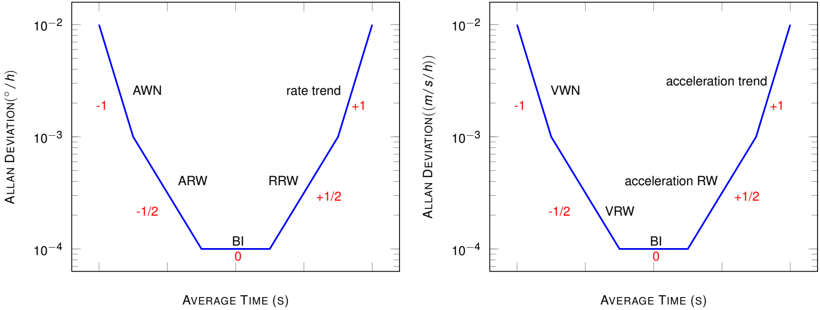
3. Plot log-log of the Allan deviation which is square root of

$$\sigma^2_{Allan}(nT_s) = \frac{1}{2(N-1)} \sum_{j=1}^{N-1} (y_j - y_{j-1})^2 \quad (10)$$

versus averaging time $\tau = nT_s$

One-sided PSD - Typical Slopes





10^{-2}
10^{-3}
10^{-4}
ALLAN DEVIATION($\circ/h$)
AWN
-1
ARW
-1/2
BI
0
RRW
+1/2
rate trend
+1
AVERAGE TIME (S)
10^{-2}
10^{-3}
10^{-4}
ALLAN DEVIATION($(m/s/h)$)
VWN
-1
VRW
-1/2
BI
0
acceleration RW
+1/2
acceleration trend
+1
AVERAGE TIME (S)

Noise Type	AV $\sigma^2(\tau)$	PSD (2-sided)
Quantization Noise	$3\frac{\alpha^2}{\tau^2}$	$(2\pi f)^2\alpha^2 T_s$
Angle/Velocity Random Walk	$\frac{\alpha^2}{\tau}$	α^2
Flicker Noise	$\frac{2\alpha^2 \ln(2)}{\pi}$	$\frac{\alpha^2}{2\pi f}$
Angular Rate/Accel Random Walk	$\frac{\alpha^2\tau}{3}$	$\frac{\alpha^2}{(2\pi f)^2}$
Ramp Noise	$\frac{\alpha^2\tau^2}{2}$	$\frac{\alpha^2}{(2\pi f)^3}$