EE 570: Location and Navigation

Gyro and Accel Noise Characteristics

Aly El-Osery

Electrical Engineering Department, New Mexico Tech
Socorro, New Mexico, USA

February 20, 2013
A constant in the output of a gyro in the absence of rotation, in $\degree/h$. 
Gyro Constant Bias ($^\circ / h$)

A constant in the output of a gyro in the absence of rotation, in $^\circ / h$.

**Error Growth**

Linearly growing error in the angle domain of $\epsilon t$. 
Gyro Constant Bias ($^\circ/\text{h}$)

A constant in the output of a gyro in the absence of rotation, in $^\circ/\text{h}$.

Error Growth

Linearly growing error in the angle domain of $\epsilon t$.

Model

Random constant.
Gyro Integrated White Noise

Assuming the rectangular rule is used for integration, a sampling period of $T_s$ and a time span of $nT_s$.

$$\int_0^t \epsilon(\tau) d\tau = T_s \sum_{i=1}^n \epsilon(t_i)$$  \hspace{1cm} (1)

since $\mathbb{E}[\epsilon(t_i)] = 0$ and $\text{Cov}(\epsilon(t_i), \epsilon(t_j)) = 0$ for all $i \neq j$, $\text{Var}[\epsilon(t_i)] = \sigma^2$

$$\mathbb{E} \left[ \int_0^t \epsilon(\tau) d\tau \right] = T_s n \mathbb{E}[\epsilon(t_i)] = 0, \forall i$$  \hspace{1cm} (2)

$$\text{Var} \left[ \int_0^t \epsilon(\tau) d\tau \right] = T_s^2 n \text{Var}[\epsilon(t_i)] = T_s t \sigma^2, \forall i$$  \hspace{1cm} (3)
Gyro Integrated White Noise

Gyro Noise Characteristics

Accel Noise Characteristics

Allan Variance

Using PSD and Allan Variance

Aly El-Osery (NMT)

EE 570: Location and Navigation

February 20, 2013
Angle Random Walk (°/√h)

Integrated noise resulted in zero-mean random walk with standard deviation that grows with time as

\[ \sigma_\theta = \sigma \sqrt{T_s t} \]  \hspace{1cm} (4)

We define ARW as

\[ ARW = \sigma_\theta (1) \text{ (°/} \sqrt{h}) \]  \hspace{1cm} (5)

In terms of PSD

\[ ARW(°/\sqrt{h}) = \frac{1}{60} \sqrt{PSD((°/h)^2/\text{Hz})} \]  \hspace{1cm} (6)
Angle Random Walk ($^\circ/\sqrt{h}$)

Integrated noise resulted in zero-mean random walk with standard deviation that grows with time as

$$\sigma_\theta = \sigma \sqrt{T_s t} \quad (4)$$

We define $ARW$ as

$$ARW = \sigma_\theta (1) \quad (^\circ/\sqrt{h}) \quad (5)$$

In terms of PSD

$$ARW(^\circ/\sqrt{h}) = \frac{1}{60} \sqrt{PSD((^\circ/h)^2/Hz)} \quad (6)$$

**Error Growth**

$ARW$ times root of the time in hours.
Angle Random Walk ($\circ / \sqrt{h}$)

Integrated noise resulted in zero-mean random walk with standard deviation that grows with time as

$$\sigma_\theta = \sigma \sqrt{T_st} \tag{4}$$

We define $ARW$ as

$$ARW = \sigma_\theta (1) \quad (\circ / \sqrt{h}) \tag{5}$$

In terms of PSD

$$ARW (\circ / \sqrt{h}) = \frac{1}{60} \sqrt{PSD((\circ / h)^2 / \text{Hz})} \tag{6}$$

**Error Growth**

$ARW$ times root of the time in hours.

**Model**

White noise.

<table>
<thead>
<tr>
<th>Gyro Noise Characteristics</th>
<th>Accel Noise Characteristics</th>
<th>Allan Variance</th>
<th>Using PSD and Allan Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aly El-Osery (NMT)</td>
<td>EE 570: Location and Navigation</td>
<td></td>
<td>February 20, 2013</td>
</tr>
</tbody>
</table>
Gyro Bias Instability ($^\circ/\text{h}$)

- Due to flicker noise with spectrum $1/F$.
- Results in random variation in the bias.
- Normally more noticeable at low frequencies.
- At high frequencies, white noise is more dominant.
Gyro Bias Instability ($^\circ$/h)

- Due to flicker noise with spectrum $1/F$.
- Results in random variation in the bias.
- Normally more noticeable at low frequencies.
- At high frequencies, white noise is more dominant.

Error Growth

Variance grows over time.
Gyro Bias Instability ($^\circ/\text{h}$)

- Due to flicker noise with spectrum $1/F$.
- Results in random variation in the bias.
- Normally more noticeable at low frequencies.
- At high frequencies, white noise is more dominant.

Error Growth

Variance grows over time.

Model

First order Gauss-Markov.
Accel Constant Bias ($\mu g$)

A constant deviation in the accelerometer from the true value, in $m/s^2$. 
A constant deviation in the accelerometer from the true value, in $m/s^2$.

**Error growth**

Double integrating a constant bias error of $\epsilon$ results in a quadratically growing error in position of $\epsilon t^2/2$. 
A constant deviation in the accelerometer from the true value, in $m/s^2$.

**Error growth**

Double integrating a constant bias error of $\epsilon$ results in a quadratically growing error in position of $\epsilon t^2 / 2$.

**Model**

Random constant.
Velocity Random Walk \((m/s/\sqrt{h})\)

Integrating accelerometer output containing white noise results in velocity random walk (VRW) \((m/s/\sqrt{h})\). Similar to development of ARW, if we double integrate white noise we get

\[
\int_0^t \int \varepsilon(\tau) \, d\tau \, d\tau = T_{s,\text{sensor}}^2 \sum_{i=1}^{n} \sum_{j=1}^{i} \varepsilon(t_j)
\]

(7)
Velocity Random Walk \((m/s/\sqrt{h})\)

Integrating accelerometer output containing white noise results in velocity random walk (VRW) \((m/s/\sqrt{h})\). Similar to development of ARW, if we double integrate white noise we get

\[
\int_0^t \epsilon(\tau) d\tau d\tau = T_{s,\text{sensor}}^2 \sum_{i=1}^{n} \sum_{j=1}^{i} \epsilon(t_j)
\]  

(7)

Error Growth

Computing the variance results in

\[
\sigma_p \approx \sigma t^{(3/2)} \sqrt{\frac{T_s}{3}}
\]  

(8)
Integrating accelerometer output containing white noise results in velocity random walk (VRW) \( (m/s/\sqrt{h}) \). Similar to development of ARW, if we double integrate white noise we get

\[
\int \int_0^t \epsilon(\tau) d\tau d\tau = T_{s, sensor}^2 \sum_{i=1}^{n} \sum_{j=1}^{i} \epsilon(t_j)
\]

(7)

**Error Growth**

Computing the variance results in

\[
\sigma_p \approx \sigma t^{(3/2)} \sqrt{\frac{T_s}{3}}
\]

(8)

**Model**

White noise.
Gyro Noise Characteristics

Accel Noise Characteristics

Allan Variance

Using PSD and Allan Variance

Aly El-Osery (NMT)

EE 570: Location and Navigation

February 20, 2013

Accel Bias Stability ($\mu g$)

Error growth

Grows as $t^{5/2}$. 
Error growth
Grows as $t^{5/2}$.

Model
First order Gauss-Markov.
Allan Variance Introduction

It is a time domain analysis techniques designed originally for characterizing noise in clocks. It was first proposed by David Allan in 1966.
Divide your N-point data sequence into adjacent windows of size $n = 1, 2, 4, 8, \ldots, M \leq N/2$.

For every $n$ generate the sequence

$$y_j(n) = \frac{x_{nj} + x_{nj+1} + \cdots + x_{nj+n-1}}{n}, \quad j = 0, 1, \ldots, \left[\frac{N}{n}\right] - 1$$  \hspace{1cm} (9)

Plot log-log of the Allan deviation which is square root of

$$\sigma_{Allan}^2(nT_s) = \frac{1}{2(N-1)} \sum_{j=1}^{N-1} (y_j - y_{j-1})^2$$  \hspace{1cm} (10)

versus averaging time $\tau = nT_s$
One-sided PSD - Typical Slopes

Gyro Noise Characteristics

Accel Noise Characteristics

Allan Variance

Using PSD and Allan Variance

Aly El-Osery (NMT)

EE 570: Location and Navigation

February 20, 2013
Allan Deviation - Typical Slopes

**Gyro Noise Characteristics**

**Accel Noise Characteristics**

**Allan Variance**

**Using PSD and Allan Variance**

Aly El-Osery (NMT)

EE 570: Location and Navigation

February 20, 2013
## Noise Parameters

<table>
<thead>
<tr>
<th>Noise Type</th>
<th>AV $\sigma^2(\tau)$</th>
<th>PSD (2-sided)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantization Noise</td>
<td>$3\frac{\alpha^2}{\tau^2}$</td>
<td>$(2\pi f)^2\alpha^2 T_s$</td>
</tr>
<tr>
<td>Angle/Velocity Random Walk</td>
<td>$\frac{\alpha^2}{\tau}$</td>
<td>$\alpha^2$</td>
</tr>
<tr>
<td>Flicker Noise</td>
<td>$\frac{2\alpha^2 \ln(2)}{\pi}$</td>
<td>$\frac{\alpha^2}{2\pi f}$</td>
</tr>
<tr>
<td>Angular Rate/Accel Random Walk</td>
<td>$\frac{\alpha^2 \tau}{3}$</td>
<td>$\frac{\alpha^2}{(2\pi f)^2}$</td>
</tr>
<tr>
<td>Ramp Noise</td>
<td>$\frac{\alpha^2 \tau^2}{2}$</td>
<td>$\frac{\alpha^2}{(2\pi f)^3}$</td>
</tr>
</tbody>
</table>