1. In class, we developed the basic (elementary) rotation matrix

$$C_{z,\theta} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

that describes the orientation of a coordinate frame rotated away from another coordinate frame by an angle θ about the $z-$axis.

   (a) Derive the basic (elementary) rotation matrix $C_{y,\theta}$ that describes the orientation of a coordinate frame rotated away from another coordinate frame by an angle θ about the $y-$axis.

   (b) Derive the basic (elementary) rotation matrix $C_{x,\theta}$ that describes the orientation of a coordinate frame rotated away from another coordinate frame by an angle θ about the $x-$axis.

2. For each of the matrices below, determine which are valid rotation matrices. Justify your answer based upon expected properties.

   (a) $C_b^a = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

   (b) $C_c^b = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

   (c) $C_d^c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$

   (d) $C_e^d = \begin{bmatrix} 0.4330 & -0.7718 & 0.4656 \\ 0.7500 & 0.5950 & 0.2888 \\ -0.5000 & 0.2241 & 0.8365 \end{bmatrix}$

   (e) $C_f^e = \begin{bmatrix} 0.5000 & -0.1464 & 0.8536 \\ 0.5000 & -0.8536 & -0.1464 \\ -0.7071 & 0.5000 & 0.5000 \end{bmatrix}$

3. Consider the rotation matrix

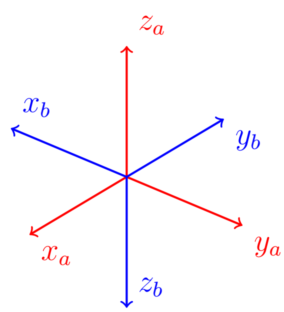
$$C_1^0 = \begin{bmatrix} 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -1 & 0 & 0 \end{bmatrix}.$$

   (a) Sketch frames 0 and 1 with their origins co-located.

   (b) Given a vector $\vec{v}^0 = [1,\ 1,\ 1]^T$ coordinatized in frame 0, re-coordinatize the vector such that it is described in frame 1.

4. For each pair of coordinate frames shown, find the rotation matrix C_b^a that describes their relative orientation.

   (a)



   (b)