1. How many multiplies and additions are needed for each of the following computations?

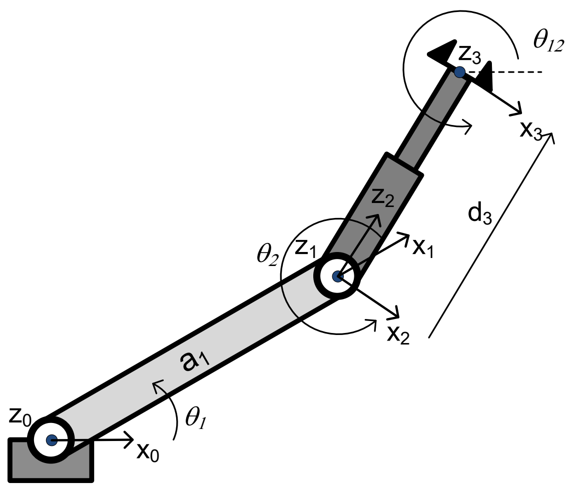
   (a) composition of rotations via rotation matrices, $C_1^2 C_0^1$

   (b) composition of rotations via quaternions, $\bar{q}_1^2 \otimes \bar{q}_0^1$

   (c) recoordinatization of a vector via rotation matrix, $C_1^2 \vec{r}^1$

   (d) recoordinatization of a vector via quaternion, $\bar{q}_1^2 \otimes \breve{r}^1 \otimes (\bar{q}_1^2)^{-1}$

2. Consider the three-link, planar robot shown below for which four coordinate frames have been assigned. Frame $\{0\}$ is fixed, frame $\{1\}$ rotates with angle θ_1 relative to frame $\{0\}$, frame $\{2\}$ rotates with angle θ_2 relative to frame $\{1\}$, and frame $\{3\}$ translates with distance d_3 relative to frame $\{2\}$.



   (a) Develop the rotation matrices and displacements between frames and show they are the following.

$$C_1^0 = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \vec{r}_{01}^{\,0} = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}, \quad C_2^1 = \begin{bmatrix} c_2 & 0 & -s_2 \\ s_2 & 0 & c_2 \\ 0 & -1 & 0 \end{bmatrix}, \quad \vec{r}_{12}^{\,1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_3^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \vec{r}_{23}^{\,2} = \begin{bmatrix} 0 \\ 0 \\ d_3 \end{bmatrix}$$

   Note the notation $c_1 = \cos(\theta_1)$ and $s_1 = \sin(\theta_1)$.

   (b) Determine the rotation matrix C_3^0.

(c) Determine the translation vector $\vec{r}^{\,0}_{03}$.

(d) Determine the following angular velocities as skew-symmetric matrices Ω and vectors $\vec{\omega}$. Note θ_1, θ_2 and d_3 can vary with time.

i. Ω^0_{01}, $\vec{\omega}^0_{01}$

ii. Ω^1_{12}, $\vec{\omega}^1_{12}$

iii. Ω^2_{23}, $\vec{\omega}^2_{23}$

iv. Ω^0_{03}, $\vec{\omega}^0_{03}$

3. Consider the time-varying coordinate transformation matrix C^n_b given below that describes the orientation of the body frame as it rotates with respect to the navigation frame.

$$C^n_b = \begin{bmatrix} \cos(t) & \sin(t)\sin(t^2) & \sin(t)\cos(t^2) \\ 0 & \cos(t^2) & -\sin(t^2) \\ -\sin(t) & \cos(t)\sin(t^2) & \cos(t)\cos(t^2) \end{bmatrix}$$

(a) Determine the analytic form of the time-derivative of C^n_b (i.e., $\dot{C}^n_b = \frac{dC^n_b}{dt}$) via a term-by-term differentiation.

(b) Develop MATLAB functions which accept "t" (i.e., time) as a numerical input and return C^n_b and $\dot{C}^n_b$, respectively, as numerical outputs.

(c) Using the C^n_b and $\dot{C}^n_b$ functions from above, compute the angular velocity vector $\vec{\omega}^n_{nb}$ at time $t = 0$ sec (**Hint**: you might want to compute Ω^n_{nb} first).

i. What is the magnitude (i.e., $\dot{\theta}$, angular speed) of the angular velocity?

ii. About what unit vector ($\vec{k}^n_{nb}$) has the instantaneous rotation occurred?

(d) Using the C^n_b and $\dot{C}^n_b$ functions from above, compute the angular velocity vector $\vec{\omega}^n_{nb}$ at time $t = 0.5$ sec.

i. What is the magnitude (i.e., $\dot{\theta}$, angular speed) of the angular velocity?

ii. About what unit vector ($\vec{k}^n_{nb}$) has the instantaneous rotation occurred?

(e) Using C^n_b and $\dot{C}^n_b$ functions from above, compute the angular velocity vector $\vec{\omega}^n_{nb}$ at time $t = 1$ sec.

i. What is the magnitude (i.e., $\dot{\theta}$, angular speed) of the angular velocity?

ii. About what unit vector ($\vec{k}^n_{nb}$) has the instantaneous rotation occurred?

(f) In practice, direct measurement of the angular velocity vector $\vec{\omega}^n_{nb}$ can prove challenging, so a finite-difference approach may be taken given two sequential orientations represented by $C^n_b(t)$ and $C^n_b(t + \Delta t)$ a small time Δt apart. Consider the approximate value of the angular velocity vector $\vec{\omega}^n_{nb}$ derived by using the finite difference

$$\dot{C}^n_b(t) \approx \frac{C^n_b(t + \Delta t) - C^n_b(t)}{\Delta t}$$

at times $t = 0, 0.5$, and 1 sec. Compare the "analytic" values for $\dot{\theta}$ and $\vec{k}^n_{nb}$ (found in parts b, c and d) with your approximations from the finite difference using $\Delta t = 0.1$ sec. How large are the errors?

(g) How small does the sampling time (i.e., Δt) need to be to get a "good" (better than 99.9%) approximation of the angular speed (i.e., $\dot{\theta}$)?