1. Given orientation of the b-frame *wrt* the n-frame in terms of relative yaw (ψ), pitch (θ), then roll (ϕ) angles

$$C_b^n = R_{(\vec{z},\psi)} R_{(\vec{y},\theta)} R_{(\vec{x},\phi)} = \begin{bmatrix} c_\psi & -s_\psi & 0 \\ s_\psi & c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi & c_\phi \end{bmatrix}$$
$$= \begin{bmatrix} c_\theta c_\psi & c_\psi s_\theta s_\phi - c_\phi s_\psi & c_\phi c_\psi s_\theta + s_\phi s_\psi \\ c_\theta s_\psi & c_\phi c_\psi + s_\theta s_\phi s_\psi & c_\phi s_\theta s_\psi - c_\psi s_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix} \tag{1}$$

   (a) Write a MATLAB function `dcm2angles` to extract ψ, θ, and ϕ from a DCM

   (b) Write a MATLAB function `q2angles` to extract ψ, θ, and ϕ from a quaternion

2. Consider the time-varying coordinate transformation matrix C_b^n given below that describes the orientation of the body frame as it rotates with respect to the navigation frame.

$$C_b^n = \begin{bmatrix} \cos(t) & \sin(t)\sin(t^2) & \sin(t)\cos(t^2) \\ 0 & \cos(t^2) & -\sin(t^2) \\ -\sin(t) & \cos(t)\sin(t^2) & \cos(t)\cos(t^2) \end{bmatrix} \tag{2}$$

   (a) Compute an expression for ψ, θ, and ϕ as defined in Equation(1)

   (b) Write MATLAB code to plot ψ, θ, and ϕ

   (c) Compute ψ, θ, and ϕ by using your `dcm2angles` function

   (d) Compute ψ, θ, and ϕ by using your `q2angles` function

   (e) Plot results from (c) and (d), and compare them to your ground truth from (b)

3. We can iteratively update the attitude using

$$\bar{\boldsymbol{q}}_b^n(+) = \bar{\boldsymbol{q}}_b^n(-) \otimes \Delta\bar{\boldsymbol{q}}_{b(k)}^{b(k-1)} \tag{3}$$

   where $\vec{\omega}_{nb}^{\,b}\Delta t = \vec{k}\Delta\theta$ and

$$\Delta\bar{\boldsymbol{q}}_{b(k)}^{b(k-1)} = \begin{bmatrix} \cos(\frac{\Delta\theta}{2}) \\ \vec{k}\sin(\frac{\Delta\theta}{2}) \end{bmatrix}$$

   Derive a quaternion update equation using $\vec{\omega}_{nb}^{\,n}$.

4. Using the DCM in Equation(2) at time $t = 0$ as your initial attitude, assume a $\delta t = 0.0005$

(a) write MATLAB code to update your attitude over time using

   i. first order approximation of derivatives

   ii. quaternions

(b) compare your results with ground truth, e.g., compute and plot the error in roll, pitch and yaw angles

(c) what is the impact of δt on your results?