

# EE 570: Location and Navigation

## Global Navigation Satellite Systems (GNSS)

### Part I

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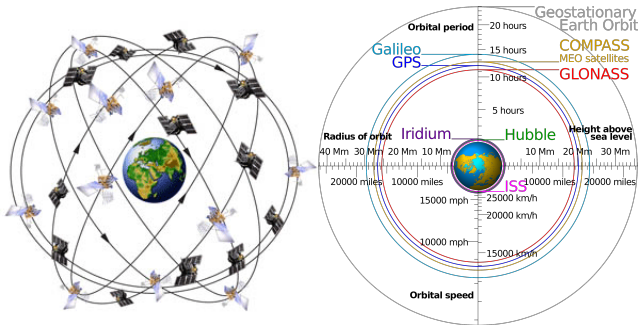
April 25, 2016

- Navigation can be accomplished via “position fixing” or “dead reckoning”
  - ① Dead Reckoning — Measures changes in position and/or attitude
    - Inertial sensors provide relative position (and attitude)
  - ② Position Fixing — Directly measuring location
    - GPS provides absolute position (and velocity)
- How does GPS work?
  - Effectively via Multilateration
    - If I can measure my distance to three (or more) satellites at known locations, then, own location can be resolved. *Measure distance via “time-of-flight”*

- GNSS — A generic term used to describe these navigation systems that provide a user with 3-D positioning solution using RF ranging of signals transmitted by orbiting satellite
- GNSS examples include
  - NAVSTAR — Navigation by Satellite Ranging and Timing operated by the United States commonly referred to as Global Positioning System (GPS)
  - GLONASS — Russian
  - Galileo — European
  - Beidou — China

- Space segment (satellites)
- Control segment
- User segment

- Collection of satellites known as *constellation*
- Broadcasts signals to control segment and the users
- Distributed among different medium Earth orbits (MEOs)
- GPS satellites
  - orbit at a radius of 26,580km
  - two orbits per sidereal day

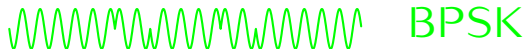
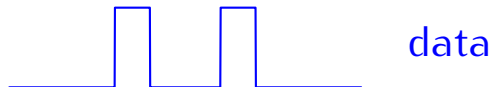
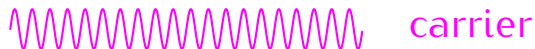


- Consists of
  - monitoring stations — at surveyed locations with synchronized clocks and collects ranging measurements
  - control stations — received data from monitoring stations and calculates corrections
  - uplink stations — sends commands to the satellites.

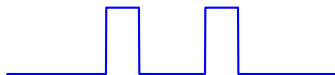
In general, a GNSS signal is a carrier with a spreading code modulated using binary phase shift keying (BPSK) given by

$$s(t) = \sqrt{2P}C(t)D(t)\cos(2\pi f_{ca}t + \phi_0) \quad (1)$$

where  $P$  is the signal power,  $C(t)$  is the spreading code,  $D(t)$  is the data,  $f_{ca}$  is the carrier frequency, and  $\phi_0$  is the phase offset.  $C(t)$  and  $D(t)$  have  $\pm 1$  values.







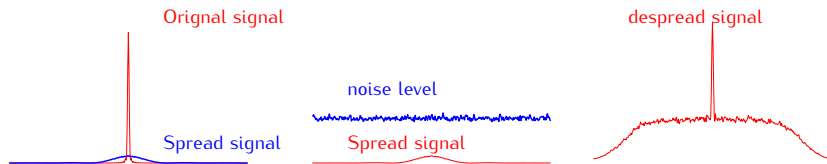
data



PRN



spread data



- The GPS employs BPSK modulation at two frequencies
  - L1=1,575.42 MHz
  - L2=1,227.60 MHz
- Two main PRN code
  - C/A: Course acquisition (10-bit 1MHz)
  - P: Precise
    - 40-bit 10MHz
    - Encrypted P(Y) code

By determining the phase of the received PRN code the raw pseudo-range to a given satellite is given by

$$\tilde{\rho}_{a,R}^s = (\tilde{t}_{sa}^s - \tilde{t}_{st,a}^s)c \quad (2)$$

where  $\tilde{t}_{sa}^s$  is the transmission time of the signal from the satellite,  $s$ ,  $\tilde{t}_{st,a}^s$  is the arrival time at antenna,  $a$ , and  $c$  is the speed of light.

The true range from an antenna  $a$  to a satellite  $s$  in the ECEF frame is given by

$$\vec{r}_{as} = |\vec{r}_{es}^e(t_{st,a}^s) - \vec{r}_{ea}^e(t_{sa,a}^s)| + \delta\rho_{ie,a}^s \quad (3)$$

where  $\delta\rho_{ie,a}^s$  is a correction factor due to rotation of the earth causing Sagnac effect. The line-of-sight unit vector (direction from which a signal arrives at the user antenna) in the ECEF frame is given by

$$\vec{u}_{as}^e \approx \frac{\vec{r}_{es}^e(t_{st,a}^s) - \vec{r}_{ea}^e(t_{sa,a}^s)}{|\vec{r}_{es}^e(t_{st,a}^s) - \vec{r}_{ea}^e(t_{sa,a}^s)|} \quad (4)$$

The range rate using ECEF velocities is

$$\dot{r}_{as} = (\vec{u}_{as}^e)^T (\vec{v}_{es}^e(t_{st,a}^s) - \vec{v}_{ea}^e(t_{sa,a}^s)) + \delta\dot{\rho}_{ie,a}^s \quad (5)$$

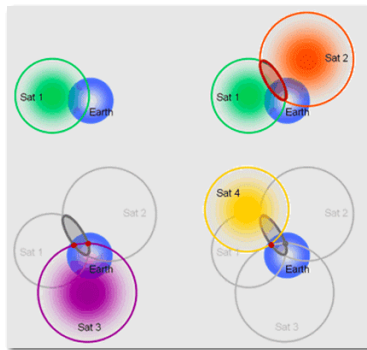
The Sagnac correction is approximated as

$$\delta \rho_{ie,a}^s \approx \frac{\omega_{ie}}{c} [y_{es}^e(t_{st,a}^s) x_{ea}^e(t_{sa,a}^s) - x_{es}^e(t_{st,a}^s) y_{ea}^e(t_{sa,a}^s)] \quad (6)$$

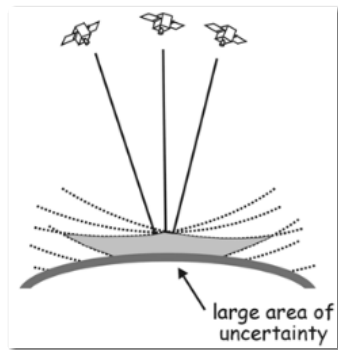
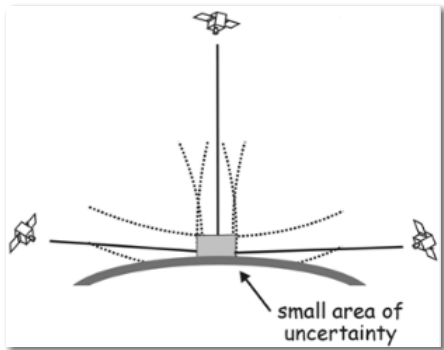
and the range-rate Sagnac correction is

$$\delta \dot{\rho}_{ie,a}^s \approx \frac{\omega_{ie}}{c} \begin{pmatrix} v_{es,y}^e(t_{st,a}^s) x_{ea}^e(t_{sa,a}^s) + y_{es}^e(t_{st,a}^s) v_{ea,x}^e(t_{sa,a}^s) \\ -v_{es,x}^e(t_{st,a}^s) y_{ea}^e(t_{sa,a}^s) - x_{es}^e(t_{st,a}^s) v_{ea,y}^e(t_{sa,a}^s) \\ 0 \end{pmatrix} \quad (7)$$

Use the range to multiple satellites to determine the position of the user equipment.



GNSS solution is affected by the geometry of the satellite constellation observed by the receiver antenna.





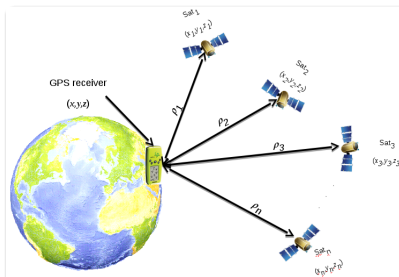
All measurements are in ECEF

$$\rho_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2}$$

$$\rho_i^2 = x_i^2 + y_i^2 + z_i^2 - 2x_i x + y_i^2 + y^2 - 2y_i y + z_i^2 + z^2 - 2z_i z$$

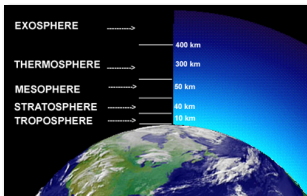
$$\rho_i^2 - (x_i^2 + y_i^2 + z_i^2) - (x^2 + y^2 + z^2) = -2x_i x - 2y_i y - 2z_i z$$

$$\begin{pmatrix} \rho_1^2 - (x_1^2 + y_1^2 + z_1^2) - r_e^2 \\ \rho_2^2 - (x_2^2 + y_2^2 + z_2^2) - r_e^2 \\ \vdots \\ \rho_n^2 - (x_n^2 + y_n^2 + z_n^2) - r_e^2 \end{pmatrix} = \begin{pmatrix} -2x_1 - 2y_1 - 2z_1 \\ -2x_2 - 2y_2 - 2z_2 \\ \vdots \\ -2x_n - 2y_n - 2z_n \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

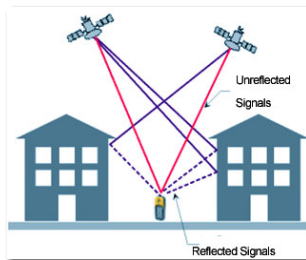


In reality there are errors in the propagation model used for the signal due to ionosphere and the troposphere. In addition there are clock errors both at the satellite and the receiver. Consequently, the pseudorange measurement is given by

$$\rho_{\text{measured}} = \rho_{\text{true}} + \epsilon_{\text{ionospheric}} + \epsilon_{\text{tropospheric}} + \epsilon_{\text{ephemeris}} + \epsilon_{\text{satellite clock}} + \epsilon_{\text{receiver clock}} + \epsilon_{\text{multipath}} \quad (8)$$

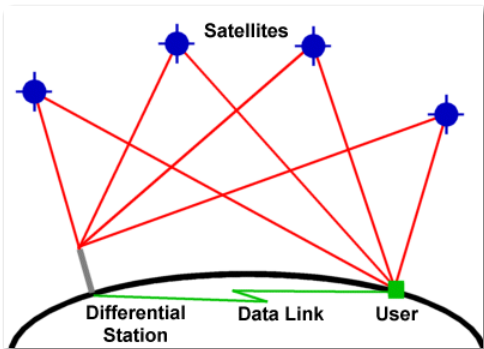


[www.intellego.fr](http://www.intellego.fr) (blog by manumanu)



<http://www.engineeringsall.com/sources-of-errors-in-gps/>

- Measure pseudorange error at surveyed locations
- Subtract error at the user equipment before calculating position



## Wide Area Augmentation System

- Provide corrections based on user position
- Assumes atmospheric errors are locally correlated.

