EE 570: Location and Navigation

Global Navigation Satellite Systems (GNSS) Part II

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Recall

- GNSS receivers compute the position through trilateration
- Pseudoranges are computed be computing the difference between the transmission time and the receive time
- Several contributors distort the measurements and require further processing to compute accurate position
  - Doppler shift,
  - oscillator drift,
  - propagation channel effects, etc

- Front-end
- Baseband processor
- Ranging processor
- Navigation processor

Assuming BPSK, the received satellite signal is given by

$$s_a(t_{sa}) = \sqrt{2P}C(t_{st})D(t_{st})\cos\left[2\pi(f_{ca} + \Delta f_{ca})t_{sa} + \phi_0\right] \tag{1}$$

where P is the carrier signal power, C is the spreading code, D is the navigation data, ϕ_0 is the phase offset, f_{ca} is the carrier frequency, and Δf_{ca} is the doppler shift which is related to the range rate by

$$\Delta f_{ca} \approx -\frac{f_{ca}}{c}\dot{\rho}_R \tag{2}$$

The receiver front-end performs

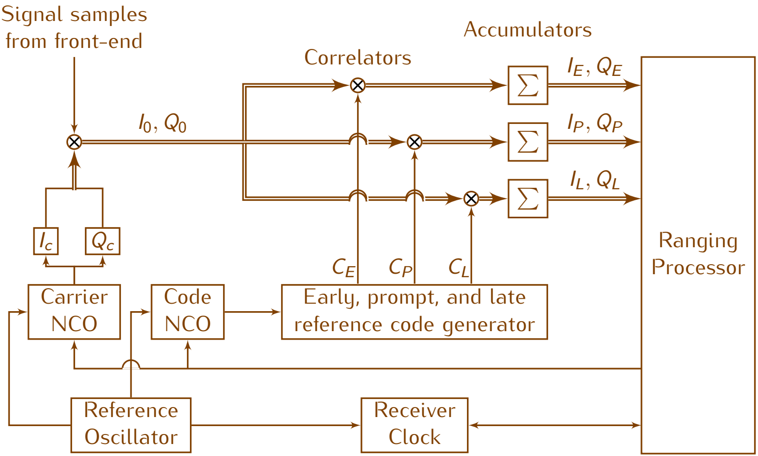
- signal conditioning,
- downconversion from L-band to IF frequency, and
- ADC

The resulting signal is of the form

$$s_{IF}(t_{sa}) = A_a C(t_{st}) D(t_{st}) \cos\left[2\pi(f_{IF} + \Delta f_{ca})t_{sa} + \phi_{IF}\right] \quad (3)$$

The baseband signal processor consists of

- parallel channels for each signal being processed,
- in-phase and quadrature sampling (allows for noncoherent processing,
- correlators, and
- accumulators.


Signal samples
from front-end
Accumulators
Correlators
I_E, Q_E
I_0, Q_0
I_P, Q_P
I_L, Q_L
I_c
Q_c
C_E
C_P
C_L
Ranging
Processor
Carrier
NCO
Code
NCO
Early, prompt, and late
reference code generator
Reference
Oscillator
Receiver
Clock

The in-phase, I_C and quadrature, Q_C are given by

$$\begin{aligned} I_C(t_{sa}) &= \cos\left[2\pi(f_{IF} + \Delta\tilde{f}_{ca})t_{sa} + \tilde{\phi}_{IF}\right] \\ Q_C(t_s a) &= \sin\left[2\pi(f_{IF} + \Delta\tilde{f}_{ca})t_{sa} + \tilde{\phi}_{IF}\right] \end{aligned} \tag{4}$$

where Δf_{ca} is the estimate of the Doppler shift from the ranging processor. The in-phase and quadrature samples are then

$$\begin{aligned} I_0(t_{sa}) &= A_0 C(t_{st}) D(t_{st}) \cos\left[2\pi(\Delta f_{ca} - \Delta\tilde{f}_{ca})t_{sa} + \phi_{IF} - \tilde{\phi}_{IF}\right] + w_{I0}(t_{sa}) \\ Q_0(t_s a) &= A_0 C(t_{st}) D(t_{st}) \sin\left[2\pi(\Delta f_{ca} - \Delta\tilde{f}_{ca})t_{sa} + \phi_{IF} - \tilde{\phi}_{IF}\right] + w_{Q0}(t_{sa}) \end{aligned} \tag{5}$$

where w_{I0} and w_{Q0} are the reciever noise terms.

- An early, late and prompt reference codes are generated and multiplied by the in-phase and quadrature samples, I_0 and Q_0.
- All of these samples are then accumulated (integrate and dump) over an interval τ_a (at least 1ms).
- The accumulation time is adjusted to maximize the S/N of the in-phase and quadrature samples while maintaining integration coherence (constant phase of integration period).

The resulting tracking errors are given by

$$
\begin{aligned}
x &= (t_{st} - \tilde{t}_{st})f_{co} \\
\delta f_{ca} &= \Delta f_{ca} - \Delta \tilde{f}_{ca} \\
\delta \phi_{ca} &= \phi_{IF} - \tilde{\phi}_{IF} + (2\pi t_{sa} - \pi\tau)\delta f_{ca}
\end{aligned} \tag{6}
$$

where x is tracking errors in chips, and f_{co} is the chip rate In terms of pseudo-range and pseudo-range rates

$$
\begin{aligned}
x &= (\tilde{\rho}_R - \rho_R)\frac{f_{co}}{c} \\
\delta f_{ca} &= (\tilde{\dot{\rho}}_R - \dot{\rho}_R)\frac{f_{ca}}{c}
\end{aligned} \tag{7}
$$

The ranging processor performs

- Acquistion — determins the initial code phase after turning on the receiver or switching to new satellite
- Code tracking — refines the measurement of the code phase and send control signals to the NCO
- Carrier phase-tracking loop or carrier frequency-tracking loop — uses the prompt in-phase and quadrature signals from the baseband processor and applies a discriminator function to track the phase of the carrier frequency, or the Doppler frequency, respectively.
- Apply corrections to pseudo-range and pseudo-range rates.

The navigation processor uses pseudo-range and pseudo-range rates to compute position and velocity of the user. This may be done using

- single-epoch, or
- filtered (using Kalman filter).

Minimize the cost function

$$J = ||\tilde{\rho} - \hat{\rho}|| \tag{8}$$

Assuming we are tracking m satellites

$$\begin{pmatrix} \hat{\vec{r}}^{\,e+}_{ea} \\ \delta\hat{\rho}^{a+}_{c} \end{pmatrix} = \begin{pmatrix} \hat{\vec{r}}^{\,e-}_{ea} \\ \delta\hat{\rho}^{a-}_{c} \end{pmatrix} + (H_G^T C^{-1} H_G)^{-1} H_G^T C^{-1} \begin{pmatrix} \tilde{\rho}^{1}_{a,C} - \hat{\rho}^{1-}_{a,C} \\ \tilde{\rho}^{2}_{a,C} - \hat{\rho}^{2-}_{a,C} \\ \vdots \\ \tilde{\rho}^{m}_{a,C} - \hat{\rho}^{m-}_{a,C} \end{pmatrix} \tag{9}$$

and for the pseudo-range rates

$$\begin{pmatrix} \hat{\vec{v}}^{\,e+}_{ea} \\ \delta\hat{\rho}^{a+}_{c} \end{pmatrix} = \begin{pmatrix} \hat{\vec{v}}^{\,e-}_{ea} \\ \delta\hat{\rho}^{a-}_{c} \end{pmatrix} + (H_G^T C^{-1} H_G)^{-1} H_G^T C^{-1} \begin{pmatrix} \tilde{\dot{\rho}}^{1}_{a,C} - \hat{\dot{\rho}}^{1-}_{a,C} \\ \tilde{\dot{\rho}}^{2}_{a,C} - \hat{\dot{\rho}}^{2-}_{a,C} \\ \vdots \\ \tilde{\dot{\rho}}^{m}_{a,C} - \hat{\dot{\rho}}^{m-}_{a,C} \end{pmatrix} \tag{10}$$

In the previous slide, $\delta\hat{\rho}^a_c$ is an estimate of the clock drift, $(-)$ and $(+)$ signs indicate previous and next iterations, $\tilde{\rho}^j_{a,C}, \tilde{\dot{\rho}}^j_{a,C}$ are the corrected pseude-range and pseudo-range rate measurements of the jth satellite, and

$$\begin{aligned}
\hat{\rho}^{j-}_{a,C} &= \sqrt{[\hat{\vec{r}}^{\,e}_{ej}(\tilde{t}^j_{st,a}) - \hat{\vec{r}}^{\,e-}_{ea}(\tilde{t}^j_{sa,a})]^T[\hat{\vec{r}}^{\,e}_{ej}(\tilde{t}^j_{st,a}) - \hat{\vec{r}}^{\,e-}_{ea}(\tilde{t}^j_{sa,a})]} + \delta\hat{\rho}^{a-}_c + \delta\hat{\rho}^{j-}_{ie,a} \\
\hat{\dot{\rho}}^{j-}_{a,C} &= (\hat{\vec{u}}^{\,e-}_{aj})^T[\hat{\vec{v}}^{\,e}_{ej}(\tilde{t}^j_{st,a}) - \hat{\vec{v}}^{\,e-}_{ea}(\tilde{t}^j_{sa,a})] + \delta\hat{\dot{\rho}}^{a-}_c + \delta\hat{\dot{\rho}}^{j-}_{ie,a}
\end{aligned} \tag{11}$$

and

$$H_G = \begin{pmatrix} -u^e_{a1,x} & -u^e_{a1,y} & -u^e_{a1,z} & 1 \\ -u^e_{a2,x} & -u^e_{a2,y} & -u^e_{a2,z} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -u^e_{am,x} & -u^e_{am,y} & -u^e_{am,z} & 1 \end{pmatrix} \tag{12}$$

Finally, C is a weighting diagonal matrix based on the variance of the measurements with respect to each satellite.

Using a Kalman filter based estimation algorithm we can utilize previous information in our estimation process resulting in smoother data which also helps reduce the impact of multipath. This will also allow for a short period of coasting when there are only three satellites being tracked.

For estimation in the ECEF frame the following states are used

$$\vec{x}^{\,e} = \begin{pmatrix} \vec{r}^{\,e}_{ea} \\ \vec{v}^{\,e}_{ea} \\ \delta\rho^a_c \\ \delta\dot{\rho}^a_c \end{pmatrix} \tag{13}$$

Using the relationship between position, and velocity and clock, and clock rates

$$F^e = \begin{pmatrix} 0_{3\times3} & \mathcal{I}_{3\times3} & 0_{3\times1} & 0_{3\times1} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times1} & 0_{3\times1} \\ 0_{1\times3} & 0_{1\times3} & 0 & 1 \\ 0_{1\times3} & 0_{1\times3} & 0 & 0 \end{pmatrix} \tag{14}$$

The measurements that are used by the Kalman filter are the pseudo-range and pseudo-range rates which are none linear function of position and velocity, respectively. Therefore after linearization we get

$$H = \left(\begin{array}{cccc} -\vec{u}_{a1}^{\,T} & 0_{1\times 3} & 1 & 0 \\ -\vec{u}_{a2}^{\,T} & 0_{1\times 3} & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ -\vec{u}_{am}^{\,T} & 0_{1\times 3} & 1 & 0 \\ \hline 0_{1\times 3} & -\vec{u}_{a1}^{\,T} & 0 & 1 \\ 0_{1\times 3} & -\vec{u}_{a2}^{\,T} & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 0_{1\times 3} & -\vec{u}_{am}^{\,T} & 0 & 1 \end{array}\right) \tag{15}$$