Lecture

Navigation Mathematics: Coordinate Frames

EE 570: Location and Navigation

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Overview

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1 Coordinate Frames

Coordinate Frames

Right-hand coordinate frame $\alpha$ has
1. origin $o^\alpha$ at which frame is located, and
2. orthonormal vectors $x^\alpha, y^\alpha, z^\alpha$ that serve as axes and indicate positive directions.
Coordinate Frames
This definition implies
\[x^\alpha \cdot x^\alpha = y^\alpha \cdot y^\alpha = z^\alpha \cdot z^\alpha = 1\]
\[x^\alpha \cdot y^\alpha = y^\alpha \cdot z^\alpha = z^\alpha \cdot x^\alpha = 0\]
\[x^\alpha \times y^\alpha = z^\alpha \]
\[y^\alpha \times z^\alpha = x^\alpha \]
\[z^\alpha \times x^\alpha = y^\alpha \]

Coordinate Frames
Coordinate frames used as means to describe position and orientation/attitude of one frame with respect to another.

2 Earth-Centered Inertial (ECI) Frame

Earth-Centered Inertial (ECI) Frame
ECI Frame

- defined as an inertial frame, i.e., it is assumed not to accelerate or rotate with respect to the universe
  - ECI will be attached to earth, but won’t spin with earth
- inertial sensors measure “inertial” motion relative to ECI frame
  - Gyroscopes measure rate of change of orientation
  - Accelerometers measure linear acceleration
- referred to as i-frame
ECI Frame

- Origin $o^i$ of ECI is located near the center of mass (center of ellipsoidal representation) of the earth.
- $z^i$-axis points along the nominal axis of rotation of the earth.
  - True north, not magnetic north!
- $x^i$-axis lies in the equatorial plane and points from the earth to the sun at the vernal (spring) equinox.
  - Defined by the intersection of the equatorial plane and the earth-sun orbital plane.
- $y^i$-axis chosen to complete right hand coordinate system (90° ahead of $x^i$ in direction of earth's rotation).

*The ECI coordinate frame does not rotate with the earth.*

3 Earth-Centered Earth-Fixed (ECEF) Frame

Earth-Centered Earth-Fixed (ECEF) Frame

- Not an inertial frame.
- Fixed with respect to the earth, i.e., attached to the earth and spins with earth.
- Referred to as $e$-frame.
ECEF Frame
- origin $o^e$ is located (nearly) at the center of the mass of the earth (co-located with ECI’s $o^i$)
- $z^e$-axis points along the nominal axis of earth’s rotation (same as ECI’s $z^i$)
- $x^e$-axis lies at the intersection of the equatorial plane and the reference meridian plane (i.e., Greenwich/Prime Meridian)
  - tied to concept of latitude and longitude
  - $x^e$ points from $o^e$ towards $0^\circ$ longitude and $0^\circ$ latitude (a little west of central Africa)
- $y^e$-axis is chosen to complete right hand coordinate system

ECEF Frame
- $z^e$-axis points along axis of earth’s rotation
- $x^e$-axis points towards zero latitude and longitude
- $y^e$-axis completes right hand coordinate system
- NMT’s (lat, long) ≈ (34.07°, −106.9°) = (34.07°, 253.1°)

Local Navigation (Nav) Frame
- typically not fixed with respect to the earth, i.e., free to move, but has specified orientation
- also called geodetic, geographic, locally level, or tangential frame
- referred to as n-frame

Nav Frame
- origin $o^n$ is located at the center of mass of the body (e.g., air, land or sea vehicle) of interest
- $z^n$-axis points "down" normal to the earth’s surface (approximately towards the center of the earth)
- $x^n$ - $y^n$ axes then constrained to lie in plane locally-level (tangential) to the earth’s surface
  - $x^n$-axis points to the north pole
  - $y^n$-axis is chosen to complete right hand coordinate system
- frame’s configuration is often referred to as the NED frame
  - $x^n$ → North, $y^n$ → East, and $z^n$ → Down
Nav Frame

- $o^n$ on (potentially moving) body
- $x^n$-axis points north
- $y^n$-axis points east
- $z^n$-axis points "down"

5 Body Frame

Body Frame

- attached to moving body (e.g., land, air or sea vehicle) and moves (position and orientation/attitude) with body
- origin $o^b$ located at the center of mass of the body (co-located with Nav frame’s $o^n$)
- $x^b$-axis points “forward” wrt moving body
- $z^b$-axis points loosely “down”
  - varies with the roll/pitch of the vehicle
- $y^b$-axis chosen to complete right hand coordinate system
- referred to as $b$-frame

Body Frame

- body frame is fixed with respect to the vehicle
- $x^b$ “forward”
- $z^b$ “down”
- $y^b$ completes right hand coordinate system ("right")
6 Other Frames

Other Frames

- Wander Azimuth Frame (alternative to the Nav frame)
  - does not always point north to avoid numerical stability problems near the poles
- Other locally level frames
  - Tangential Frame
    - typically, refers to another type of the ECEF frame fixed to the Earth’s surface (not moving like the \( n \)-frame)
  - Computer Frame
    - virtual coordinate frame that represents where we think that we are

The End