

Lecture

Navigation Mathematics: Coordinate Frames

EE 570: Location and Navigation

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Overview

Contents

1	Frames	1
2	ECI	2
3	ECEF	3
4	Nav	4
5	Body	5
6	Other	6

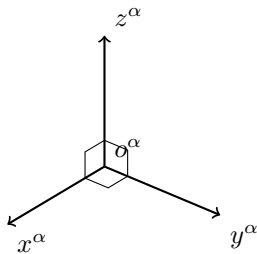
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1 Coordinate Frames

Coordinate Frames

Right-hand coordinate frame α has

1. origin o^α at which frame is located, and
2. orthonormal vectors $x^\alpha, y^\alpha, z^\alpha$ that serve as axes and indicate positive directions.



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Coordinate Frames

This definition implies

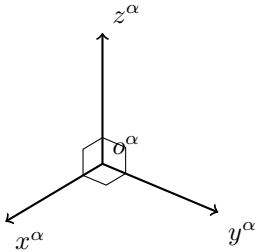
$$x^\alpha \cdot x^\alpha = y^\alpha \cdot y^\alpha = z^\alpha \cdot z^\alpha = 1$$

$$x^\alpha \cdot y^\alpha = y^\alpha \cdot z^\alpha = z^\alpha \cdot x^\alpha = 0$$

$$x^\alpha \times y^\alpha = z^\alpha$$

$$y^\alpha \times z^\alpha = x^\alpha$$

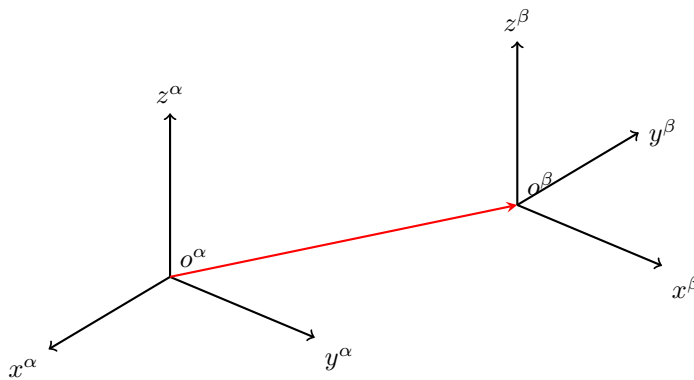
$$z^\alpha \times x^\alpha = y^\alpha$$



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Coordinate Frames

Coordinate frames used as means to describe position and orientation/attitude of one frame with respect to another.



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2 Earth-Centered Inertial (ECI) Frame

Earth-Centered Inertial (ECI) Frame

ECI Frame

- defined as an inertial frame, i.e., it is assumed not to accelerate or rotate with respect to the universe
 - ECI will be attached to earth, but won't spin with earth
- inertial sensors measure "inertial" motion relative to ECI frame
 - Gyroscopes measure rate of change of orientation
 - Accelerometers measure linear acceleration
- referred to as *i*-frame

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ECI Frame

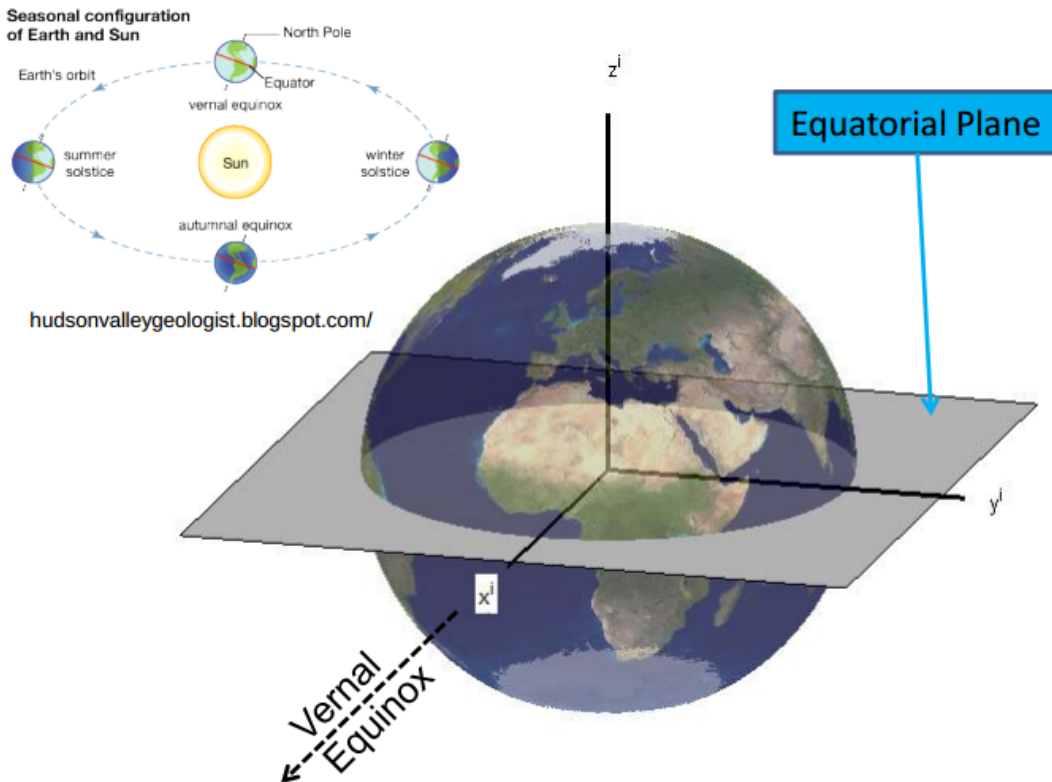
- origin o^i of ECI is located near the center of mass (center of ellipsoidal representation) of the earth
- z^i -axis points along the nominal axis of rotation of the earth
 - true north **not** magnetic north!
- x^i -axis lies in the equatorial plane and points from the earth to the sun at the vernal (spring) equinox
 - defined by the intersection of the equatorial plane and the earth-sun orbital plane
- y^i -axis chosen to complete right hand coordinate system (90° ahead of x^i in direction of earth's rotation)

*The ECI coordinate frame does **not** rotate with the earth*

.7

ECI Frame

- o^i at earth's center
- z^i -axis points along the earth's axis of rotation
- x^i -axis points towards sun at vernal (spring) equinox
- y^i -axis completes a right hand coordinate system



.8

3 Earth-Centered Earth-Fixed (ECEF) Frame

Earth-Centered Earth-Fixed (ECEF) Frame

ECEF Frame

- **not** an inertial frame
- fixed with respect to the earth, i.e., attached to the earth and spins with earth
- referred to as *e*-frame

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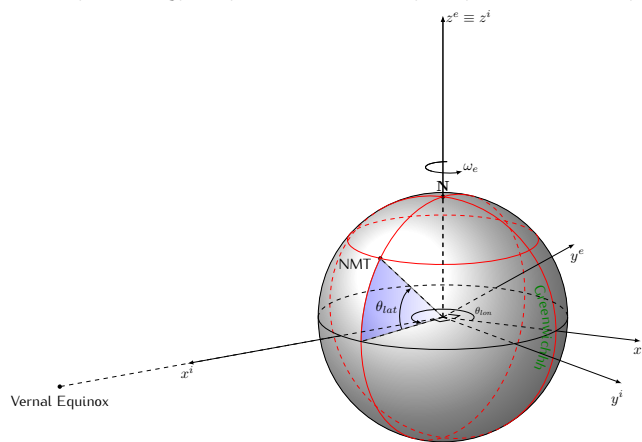
ECEF Frame

- origin o^e is located (nearly) at the center of the mass of the earth (co-located with ECI's o^i)
- z^e -axis points along the nominal axis of earth's rotation (same as ECI's z^i)
- x^e -axis lies at the intersection of the equatorial plane and the reference meridian plane (i.e., Greenwich/Prime Meridian)
 - tied to concept of latitude and longitude
 - x^e points from o^e towards 0° longitude and 0° latitude (a little west of central Africa)
- y^e -axis is chosen to complete right hand coordinate system

.10

ECEF Frame

- z^e -axis points along axis of earth's rotation
- x^e -axis points towards zero latitude and longitude
- y^e -axis completes right hand coordinate system
- NMT's (lat, long) $\approx (34.07^\circ, -106.9^\circ) = (34.07^\circ, 253.1^\circ)$



.11

4 Local Navigation (Nav) Frame

Local Navigation (Nav) Frame

Nav Frame

- typically **not** fixed with respect to the earth, i.e., free to move, but has specified orientation
- also called geodetic, geographic, locally level, or tangential frame
- referred to as n -frame

.12

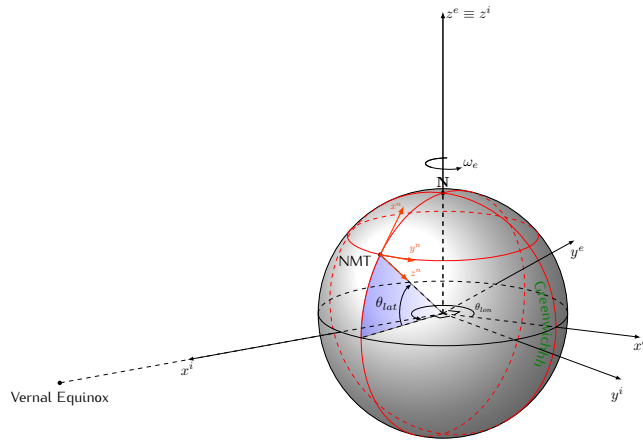
Nav Frame

- origin o^n is located at the center of mass of the body (e.g., air, land or sea vehicle) of interest
- z^n -axis points "down" normal to the earth's surface (approximately towards the center of the earth)
- $x^n - y^n$ axes then constrained to lie in plane locally-level (tangential) to the earth's surface
 - x^n -axis points to the north pole
 - y^n -axis is chosen to complete right hand coordinate system
- frame's configuration is often referred to as the NED frame
 - $x^n \rightarrow$ North, $y^n \rightarrow$ East, and $z^n \rightarrow$ Down

.13

Nav Frame

- o^n on (potentially moving) body
- x^n -axis points north
- y^n -axis points east
- z^n -axis points "down"



.14

5 Body Frame

Body Frame

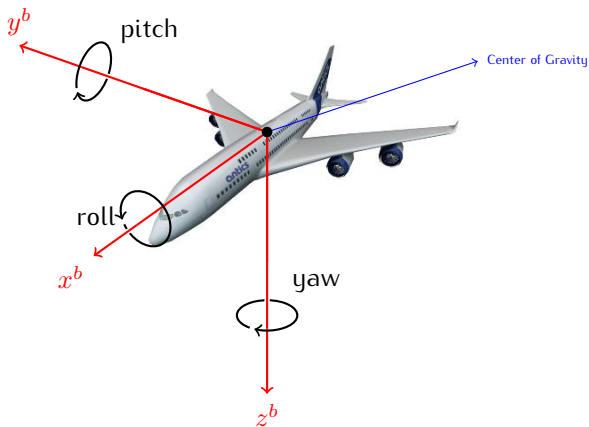
Body Frame

- attached to moving body (e.g., land, air or sea vehicle) and moves (position and orientation/attitude) with body
- origin o^b located at the center of mass of the body (co-located with Nav frame's o^n)
- x^b -axis points "forward" wrt moving body
- z^b -axis points loosely "down"
 - varies with the roll/pitch of the vehicle
- y^b -axis chosen to complete right hand coordinate system
- referred to as b -frame

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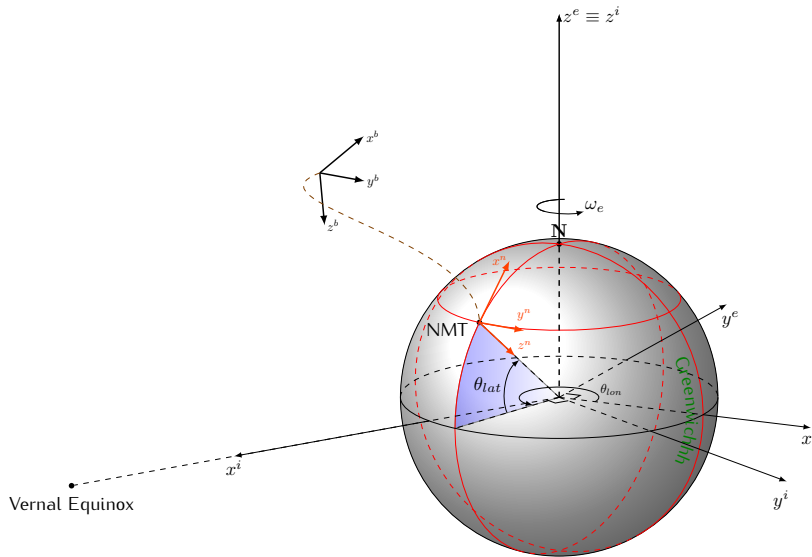
Body Frame

- body frame is fixed with respect to the vehicle
- x^b "forward"
- z^b "down"
- y^b completes right hand coordinate system ("right")



.16

Body Frame



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6 Other Frames

Other Frames

- Wander Azimuth Frame (alternative to the Nav frame)
 - does not always point north to avoid numerical stability problems near the poles
- Other locally level frames
 - Tangential Frame
 - * typically, refers to another type of the ECEF frame fixed to the Earth's surface (not moving like the n -frame)
 - Computer Frame
 - * virtual coordinate frame that represents where we think that we are

.18

The End

.19