Lecture
Gyro and Accel Noise Characteristics
EE 570: Location and Navigation

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.1

1 Inertial Sensors Errors

Inertial Sensors — Sensor Models

- Accelerometer model

$$\tilde{\vec{f}}^b_{ib} = \vec{f}^b_{ib} + \Delta\vec{f}^b_{ib} = \vec{b}_a + (\mathcal{I} + M_a)\vec{f}^b_{ib} + \vec{w}_a \tag{1}$$

- Gyro Model

$$\tilde{\vec{\omega}}^b_{ib} = \vec{\omega}^b_{ib} + \Delta\vec{\omega}^b_{ib} = \vec{b}_g + (\mathcal{I} + M_g)\vec{\omega}^b_{ib} + G_g\vec{f}^b_{ib} + \vec{w}_g \tag{2}$$

- Typically, each measures along a signle sense axis requiring three of each to measure the 3-tupple vector
- Bias errors are composite of fixed bias, bias instability, and bias stability

$$b = b_{FB} + b_{BI} + b_{BS}$$

.2

2 Gyro Noise Characteristics

Gyro Constant Bias ($°/h$)

A constant in the output of a gyro in the absence of rotation, in $°/h$.

Error Growth

Linearly growing error in the angle domain of ϵt.

Model

Random constant.

.3

Gyro Integrated White Noise

Assuming the rectangular rule is used for integration, a sampling period of T_s and a time span of nT_s.

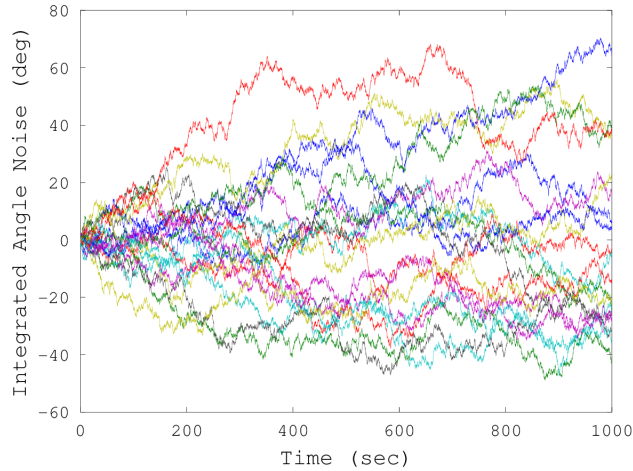$$\int_0^t \epsilon(\tau)d\tau = T_s \sum_{i=1}^n \epsilon(t_i) \tag{3}$$

since $\mathbb{E}[\epsilon(t_i)] = 0$ and $Cov(\epsilon(t_i), \epsilon(t_j)) = 0$ for all $i \neq j$, $Var[\epsilon(t_i)] = \sigma^2$

$$\mathbb{E}\left[\int_0^t \epsilon(\tau)d\tau\right] = T_s n\mathbb{E}[\epsilon(t_i)] = 0, \forall i \tag{4}$$

$$Var\left[\int_0^t \epsilon(\tau)d\tau\right] = T_s^2 nVar[\epsilon(t_i)] = T_s t\sigma^2, \forall i \tag{5}$$

Gyro Integrated White Noise

Angle Random Walk ($°/\sqrt{h}$)

Integrated noise resulted in zero-mean random walk with standard deviation that grows with time as

$$\sigma_\theta = \sigma\sqrt{T_s t} \tag{6}$$

We define ARW as

$$ARW = \sigma_\theta(1) \qquad (°/\sqrt{h}) \tag{7}$$

In terms of PSD

$$ARW(°/\sqrt{h}) = \frac{1}{60}\sqrt{PSD((°/h)^2/Hz)} \tag{8}$$

Error Growth
ARW times root of the time in hours.

Model
White noise.

Gyro Bias Instability ($°/h$)

- Due to flicker noise with spectrum $1/F$.
- Results in random variation in the bias.
- Normally more noticeable at low frequencies.
- At high frequencies, white noise is more dominant.

Error Growth
Variance grows over time.

Model
First order Gauss-Markov.

3   Accel Noise Characteristics

Accel Constant Bias (μg)
A constant deviation in the accelerometer from the true value, in m/s^2.

Error growth
Double integrating a constant bias error of ϵ results in a quadratically growing error in position of $\epsilon t^2/2$.

Model
Random constant.

Velocity Random Walk ($m/s/\sqrt{h}$)
Integrating accelerometer output containing white noise results in velocity random walk (VRW) $(m/s/\sqrt{h})$. Similar to development of ARW, if we double integrate white noise we get

$$\iint_0^t \epsilon(\tau)d\tau d\tau = T_{s,sensor}^2 \sum_{i=1}^{n}\sum_{j=1}^{i} \epsilon(t_j) \tag{9}$$

Error Growth
Computing the standard deviation results in

$$\sigma_p \approx \sigma t^{(3/2)}\sqrt{\frac{T_s}{3}} \tag{10}$$

Model
White noise.

Accel Bias Instability (μg)

Error growth
Grows as $t^{5/2}$.

Model
First order Gauss–Markov.

4   Allan Variance

Allan Variance Introduction
It is a time domain analysis techniques designed originally for characterizing noise in clocks. It was first proposed by David Allan in 1966.

Allan Variance Computation
1. Divide your N–point data sequence into adjacent windows of size $n = 1, 2, 4, 8, \ldots, M \leq N/2$.
2. For every n generate the sequence

$$y_j(n) = \frac{x_{nj} + x_{nj+1} + \cdots + x_{nj+n-1}}{n}, \quad j = 0, 1, \ldots, \left[\frac{N}{n}\right] - 1 \tag{11}$$

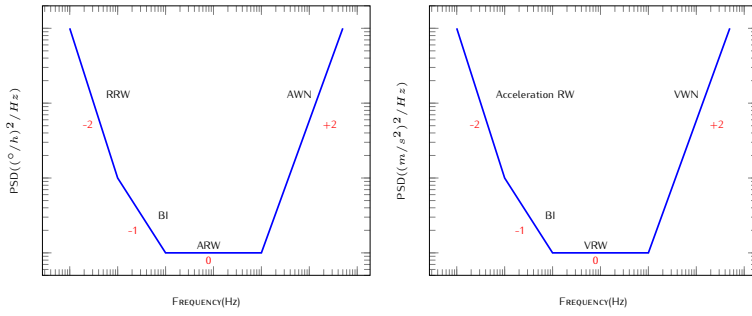3. Plot log-log of the Allan deviation which is square root of

$$\sigma_{Allan}^2(nT_s) = \frac{1}{2(N-1)}\sum_{j=1}^{N-1}(y_j - y_{j-1})^2 \tag{12}$$

versus averaging time $\tau = nT_s$

3

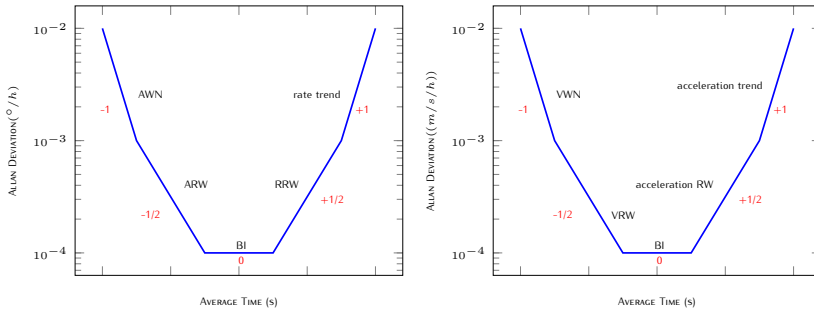# 5   Using PSD and Allan Variance

One-sided PSD - Typical Slopes

Allan Deviation - Typical Slopes

Noise Parameters

Noise Type	AV $\sigma^2(\tau)$	PSD (2-sided)
Quantization Noise	$3\frac{\alpha^2}{\tau^2}$	$(2\pi f)^2 \alpha^2 T_s$
Angle/Velocity Random Walk	$\frac{\alpha^2}{\tau}$	α^2
Flicker Noise	$\frac{2\alpha^2 \ln(2)}{\pi}$	$\frac{\alpha^2}{2\pi f}$
Angular Rate/Accel Random Walk	$\frac{\alpha^2 \tau}{3}$	$\frac{\alpha^2}{(2\pi f)^2}$
Ramp Noise	$\frac{\alpha^2 \tau^2}{2}$	$\frac{\alpha^2}{(2\pi f)^3}$