EE 565: Position, Navigation and Timing

Gyro and Accel Noise Characteristics

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March 26, 2018

Inertial Sensors — Sensor Models

- Accelerometer model

$$\tilde{\vec{f}}^{\,b}_{ib} = \vec{f}^{\,b}_{ib} + \Delta\vec{f}^{\,b}_{ib} = \vec{b}_a + (\mathcal{I} + M_a)\vec{f}^{\,b}_{ib} + \vec{w}_a \quad (1)$$

- Gyro Model

$$\tilde{\vec{\omega}}^{\,b}_{ib} = \vec{\omega}^{\,b}_{ib} + \Delta\vec{\omega}^{\,b}_{ib} = \vec{b}_g + (\mathcal{I} + M_g)\vec{\omega}^{\,b}_{ib} + G_g\vec{f}^{\,b}_{ib} + \vec{w}_g \quad (2)$$

- Typically, each measures along a signle sense axis requiring three of each to measure the 3-tupple vector
- Bias errors are composite of fixed bias, bias instability, and bias stability

$$b = b_{FB} + b_{BI} + b_{BS}$$

Gyro Constant Bias ($^\circ/h$)

A constant in the output of a gyro in the absence of rotation, in $^\circ/h$.

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Error Growth

Linearly growing error in the angle domain of ϵt.

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Model

Random constant.

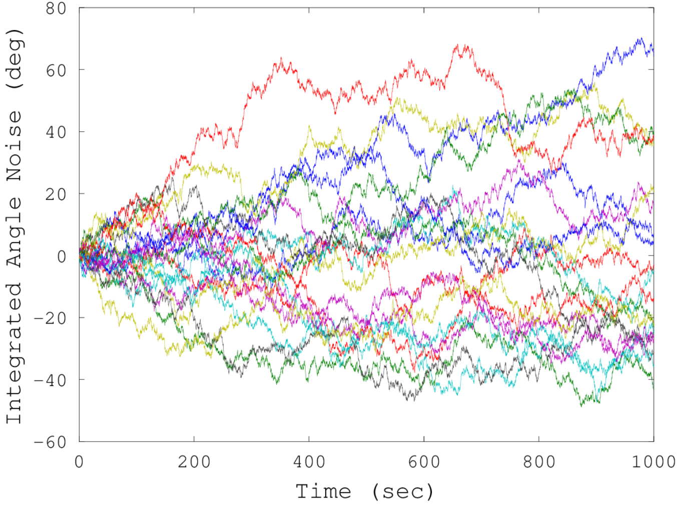
Assuming the rectangular rule is used for integration, a sampling period of T_s and a time span of nT_s.

$$\int_0^t \epsilon(\tau)d\tau = T_s \sum_{i=1}^{n} \epsilon(t_i) \tag{3}$$

since $\mathbb{E}[\epsilon(t_i)] = 0$ and $Cov(\epsilon(t_i), \epsilon(t_j)) = 0$ for all $i \neq j$, $Var[\epsilon(t_i)] = \sigma^2$

$$\mathbb{E}\left[\int_0^t \epsilon(\tau)d\tau\right] = T_s n\mathbb{E}[\epsilon(t_i)] = 0, \forall i \tag{4}$$

$$Var\left[\int_0^t \epsilon(\tau)d\tau\right] = T_s^2 nVar[\epsilon(t_i)] = T_s t\sigma^2, \forall i \tag{5}$$


Integrated Angle Noise (deg)
Time (sec)
80
60
40
20
0
−20
−40
−60
0
200
400
600
800
1000

Integrated noise resulted in zero-mean random walk with standard deviation that grows with time as

$$\sigma_\theta = \sigma\sqrt{T_s t} \tag{6}$$

We define *ARW* as

$$ARW = \sigma_\theta(1) \qquad (^\circ/\sqrt{h}) \tag{7}$$

In terms of PSD

$$ARW(^\circ/\sqrt{h}) = \frac{1}{60}\sqrt{PSD((^\circ/h)^2/Hz)} \tag{8}$$

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Model

White noise.

Gyro Bias Instability ($^\circ/h$)

- Due to flicker noise with spectrum $1/F$.
- Results in random variation in the bias.
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- At high frequencies, white noise is more dominant.

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Model

First order Gauss-Markov.

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A constant deviation in the accelerometer from the true value, in m/s^2.

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Model

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Integrating accelerometer output containing white noise results in velocity random walk (VRW) ($m/s/\sqrt{h}$). Similar to development of ARW, if we double integrate white noise we get

$$\iint_0^t \epsilon(\tau)d\tau d\tau = T_{s,sensor}^2 \sum_{i=1}^{n} \sum_{j=1}^{i} \epsilon(t_j) \tag{9}$$

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Grows as $t^{5/2}$.

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It is a time domain analysis techniques designed originally for characterizing noise in clocks. It was first proposed by David Allan in 1966.

1. Divide your N-point data sequence into adjacent windows of size $n = 1, 2, 4, 8, \ldots, M \leq N/2$.
2. For every n generate the sequence

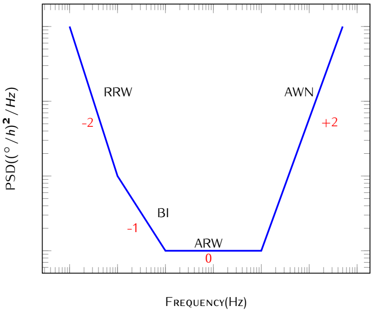
$$y_j(n) = \frac{x_{nj} + x_{nj+1} + \cdots + x_{nj+n-1}}{n}, \quad j = 0, 1, \ldots, \left\lfloor \frac{N}{n} \right\rfloor - 1 \tag{11}$$

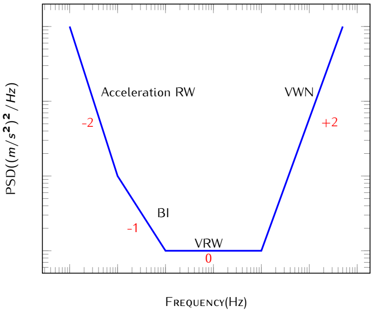
3. Plot log-log of the Allan deviation which is square root of

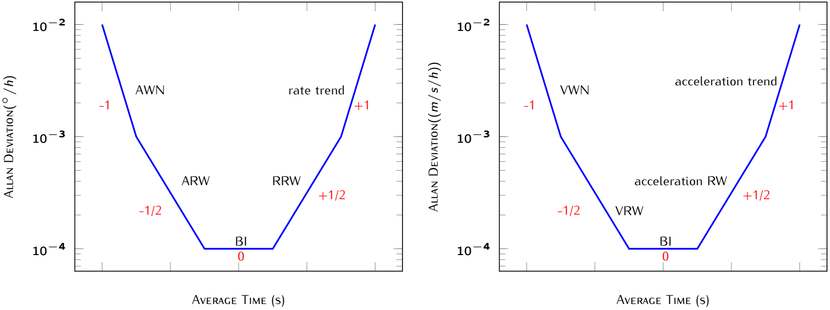
$$\sigma^2_{Allan}(nT_s) = \frac{1}{2(N-1)} \sum_{j=1}^{N-1} (y_j - y_{j-1})^2 \tag{12}$$

versus averaging time $\tau = nT_s$

One-sided PSD - Typical Slopes







10^{-2}
10^{-3}
10^{-4}
Allan Deviation(°/h)
AWN
-1
rate trend
+1
ARW
-1/2
RRW
+1/2
BI
0
Average Time (s)
10^{-2}
10^{-3}
10^{-4}
Allan Deviation((m/s/h))
VWN
-1
acceleration trend
+1
acceleration RW
+1/2
-1/2
VRW
BI
0
Average Time (s)

Noise Type	AV $\sigma^2(\tau)$	PSD (2-sided)
Quantization Noise	$3\frac{\alpha^2}{\tau^2}$	$(2\pi f)^2\alpha^2 T_s$
Angle/Velocity Random Walk	$\frac{\alpha^2}{\tau}$	α^2
Flicker Noise	$\frac{2\alpha^2 \ln(2)}{\pi}$	$\frac{\alpha^2}{2\pi f}$
Angular Rate/Accel Random Walk	$\frac{\alpha^2\tau}{3}$	$\frac{\alpha^2}{(2\pi f)^2}$
Ramp Noise	$\frac{\alpha^2\tau^2}{2}$	$\frac{\alpha^2}{(2\pi f)^3}$