

1. Write the matrix product that will give the resulting rotation matrix for the following sequence of rotations. Do not perform multiplication.
 - (a) Rotate by α about fixed x -axis
 - (b) Rotate by β about current z -axis
 - (c) Rotate by γ about fixed y -axis
 - (d) Rotate by ϕ about current y -axis
 - (e) Rotate by ψ about fixed z -axis
 - (f) Rotate by θ about current x -axis

2. Coordinate frame $\{3\}$ is obtained from frame $\{0\}$ by the following sequence of rotations:
 - (a) -90° about the fixed z -axis
 - (b) 90° about the current y -axis
 - (c) -90° about the fixed x -axis.

Find the resulting rotation matrix C_3^0 and sketch frames $\{0\}$ and $\{3\}$ relative to each other.

3. Consider the rotation matrix

$$C_b^a = \begin{bmatrix} -0.3536 & 0.1268 & -0.9268 \\ 0.6124 & 0.7803 & -0.1268 \\ 0.7071 & -0.6124 & -0.3536 \end{bmatrix}.$$

- (a) What are the Roll-Pitch-Yaw angles (ϕ, θ, ψ) that describe the orientation of frame $\{b\}$ relative to $\{a\}$?
 - (b) What is the angle-axis (θ, \vec{k}) that describes the orientation of frame $\{b\}$ relative to $\{a\}$?
 - (c) What is the quaternion \bar{q} that describes the orientation of frame $\{b\}$ relative to $\{a\}$?
 - (d) Given a velocity as the vector $\vec{v}^a = [-1, 1, -1]^T$ written in terms of frame $\{a\}$, what is the vector \vec{v}^b that describes the same velocity relative to frame $\{b\}$?
4. Given the Roll-Pitch-Yaw angles $(\phi, \theta, \psi) = (120^\circ, 45^\circ, -120^\circ)$, find the rotation matrix that describes the same orientation.
 5. Given the angle-axis $(\theta, \vec{k}) = (90^\circ, \frac{1}{\sqrt{3}}[1, 1, 1]^T)$, find the rotation matrix that describes the same orientation.

6. Given the quaternion $\bar{q} = [0.5, -0.5, -0.5, -0.5]^T$, find the rotation matrix that describes the same orientation.
7. Consider the three coordinate frames $\{\alpha\}$, $\{\beta\}$, and $\{\gamma\}$ shown in the diagram below. Following the notation introduced in class, find the following Cartesian position vectors (denoted by \vec{r}) and rotation/cosine matrices (denoted by C).

- (a) $\vec{r}_{\gamma\alpha}^\gamma$
- (b) $\vec{r}_{\alpha\gamma}^\gamma$
- (c) $\vec{r}_{\gamma\beta}^\gamma$
- (d) $\vec{r}_{\alpha\gamma}^\alpha$
- (e) $\vec{r}_{\gamma\beta}^\beta$
- (f) C_α^γ
- (g) C_β^γ
- (h) C_β^α

